

**WCER Working Paper No. 2005-10**

**October 2005**

---

## **Visual Display and Co-Expressivity as Students Strive for Intersubjectivity in a Spatial Reasoning Task**

**Mitchell J. Nathan**

Department of Educational Psychology/  
Wisconsin Center for Education Research  
University of Wisconsin–Madison  
[mnathan@wisc.edu](mailto:mnathan@wisc.edu)

**Billie Eilam**

Department of Teaching and Teacher Education  
University of Haifa, Israel  
[beilam@construct.haifa.ac.il](mailto:beilam@construct.haifa.ac.il)



**Wisconsin Center for Education Research**

**School of Education • University of Wisconsin–Madison • <http://www.wcer.wisc.edu/>**

Copyright © 2005 by Mitchell J. Nathan and Billie Eilam  
All rights reserved.

Readers may make verbatim copies of this document for noncommercial purposes by any means, provided that the above copyright notice appears on all copies.

WCER working papers are available on the Internet at  
<http://www.wcer.wisc.edu/publications/workingPapers/index.php>.

The research reported in this paper was supported by Interagency Education Research Initiative (IERI; a collaboration of NSF, Department of Education-OERI/IES, and NIH-NICHHD) under REC-0115609, entitled, “Understanding and Cultivating the Transition from Arithmetic to Algebraic Reasoning,” and by the Wisconsin Center for Education Research, School of Education, University of Wisconsin–Madison. Any opinions, findings, or conclusions expressed in this paper are those of the authors and do not necessarily reflect the views of the funding agencies, WCER, or cooperating institutions.

# Visual Display and Co-Expressivity as Students Strive for Intersubjectivity in a Spatial Reasoning Task

Mitchell J. Nathan and Billie Eilam

## Objectives and Theoretical Framework

External graphical representations enjoy certain advantages: They tend to reduce memory load (Chandler & Sweller, 1991; Sweller, 1988), facilitate information processes such as search and inference making, and allow for inspection and deictic reference (e.g., Ainsworth, 1999; Larkin & Simon, 1987; Mayer, 1999, 2003; Mayer, Heiser, & Lonn, 2001; Mayer & Moreno, 2000; Tversky, 2001). Because of their external nature, graphical representations can support cognition, communication, and collaborative problem solving in ways that internal, mental representations and ephemeral representations such as speech and gesture do not. However, participants in a discourse that involves these representations must achieve *intersubjectivity*—a shared meaning.

Engle (1998) studied the communication between undergraduate dyads about the workings of a lock-and-key system. She found that explanations included diagrams, objects, gesture, and speech that were integrated into *composite signals* rather than independently interpretable channels. Nonverbal communicative events about the locks tended to occur at the same time or nearby speech acts and tended to be co-expressive (McNeill, 1997) with neighboring speech. The composite signal view hinges on two critical assumptions. The first is that speakers and listeners share a communicative norm (somewhat like Gricean maxims) that the modalities that make up the composite signals are intended to be taken as unified. Thus, there should be evidence of co-expressivity for neighboring spoken and nonverbal behavior about the task. The second assumption is that the speaker is sufficiently skilled to provide the proper content within each modality (drawing, speech, etc.) so that a composite signal supports a single, consistent interpretation that matches the speaker's intention. Engle predicted that when speakers failed to do this, there would be unsuccessful communication, as evidenced by listener clarification requests, misunderstandings, and even speaker conversational repairs (Seedhouse, 2004).

In this study, we set out to understand whether and how intersubjectivity is sought when students in a middle school mathematics classroom all work to solve and discuss a spatial reasoning task, and what speakers do in response to failures of intersubjectivity. In particular, we set out to understand how students strive to enhance the fidelity of their communication by using co-expressive modes of communication and conversational repair, as suggested by Engle (1998).

## Data and Methods

Sixth-grade students in a middle-class community engaged in efforts to solve the Pie Problem as posed by a peer: "How do you cut a pie into eight equal-size pieces making only three cuts?" Students posed questions, presented drawn solutions, critiqued one another's approaches, and actively engaged in a rich discourse aimed at addressing this question. We used conversation analysis (Schegloff, Jefferson, & Sacks, 1977) to understand the discourse in terms

## Visual Display and Co-Expressivity

of its sequence of turns. The videotaped discourse was divided into three *phases* that reflected the different activities that occurred over time. During Phase 1, students worked by themselves to come up with their own thoughts and solutions. During this time, students generated many clarification requests and questions regarding the rules (e.g., “How about you take a pie that’s already cut, like, six times, and you cut it?”), and asked for hints (“Can you give another hint without giving it totally away, without just saying this is what you do?”). In Phase 2, students presented their individual solutions to the class using speech, drawings, and gestures. In Phase 3, students critiqued and edited solutions—using speech, gesture, drawing, erasing, and physical models—to produce a solution that was meant to be consistent with the rules and convincing to their peers. Phases were divided into *episodes* that encompassed the consecutive turns taken as they referred to a specific solution representation (typically a drawing).

For this paper, we focus our analyses on students’ attempts to achieve intersubjectivity—a common understanding of a solution to the Pie Problem. We analyzed episodes employing a coding system that highlighted students’ uses of external representations and speech to convey their solutions; captured their interpretations of representations; identified uses of co-expressive communication acts; and signaled attempts, successes, and failures at achieving intersubjectivity. We used these codes to collect evidence about (a) whether speakers sought to achieve intersubjectivity in their attempts to communicate solutions that were meaningful and persuasive to others, (b) how failure to achieve intersubjectivity resulted in misinterpretations or clarification requests by listeners, (c) how students responded to intersubjectivity failures, and (d) how co-expressive communication of neighboring communicative acts (speech, drawing, and gesture) within a given episode were used.

## Results and Conclusions

From our analyses, it was clear that the need to achieve intersubjectivity among student solutions was great. For example, students would seek validation of the accuracy of their drawings (“What’s wrong with mine?”) even if they had a geometrically legitimate solution.

Students made frequent clarification requests (“I have a question: Is that the top, the top view of the pie?”), and there were frequent attempts to explain or correct representations by the original artist or other class peers. However, three major obstacles to achieving intersubjectivity were evident: literal interpretations of representations, impoverished drawing ability, and the lack of established conventions for interpreting two-dimensional (2D) drawings of a 3D situation. In response to each of these obstacles, students tended to redouble their efforts at representing their solutions, often by providing additional, co-expressive modes of communication.

It was clear from the discourse that some students took the problem to refer to a literal pie, whereas others were willing to suspend pragmatic considerations and move on to a more abstract, geometric interpretation that would apply to any 3D object.

### **Geometric view:**

S: I’ll write there. We’ll just make the pie a square. Here’s the pie with no sides. Now this is the top view, and you cut it like that. The top view of the pie. This is the side view of the pie, and

## Visual Display and Co-Expressivity

then you cut that in half, so that was . . . and then that equals two pieces with four [indecipherable] in each of them.

### Literal view:

S: Yeah, yeah . . . if you have the top crust and you, like, lift it up, all the stuff's gonna fall out, and what if it's an apple pie. How could you . . .

The literal view made it difficult to establish intersubjectivity with those who had achieved a geometric view because the literal view challenged a central constraint of the problem—that the resulting pieces would be *equal*. Those who saw a real pie did not see equality among pieces with different kinds of crust and so could not accept the solutions offered by those who had a geometric view that made no such distinction.

In a similar fashion, some of those with a geometric view had trouble seeing the differences in crust as important.

S: Well, like, before he was saying, like . . . well, then who's going to eat a pie with just the bottom and I mean just the top and I mean this is just the example. It's just a diagram. I mean nobody's just going to come up here and eat the dry erase board.

In an attempt to achieve intersubjectivity, some students tried to articulate the literal view but reframe the problem in a context (a cake) that allowed the geometric solution to make sense.

S: But if you just make the cut, like, let's say it's a cake instead of a pie. Then it doesn't get the gooey filling and stuff. So, like, if you cut it in half, you don't have to separate the two, you just keep it stacked on top of each other and then you make the . . . two cuts on top

The second and third obstacles to achieving intersubjectivity were the limitation of students' drawing abilities and the lack of conventions for interpreting the drawn solution representations. Drawings and interpretations that violated principles of perspective led to confusion and misinterpretations of the artist's intended message. One of the most acute ways this played out is illustrated in Figure 1. A solution that had only three drawn lines representing cuts was inaccurately interpreted as producing eight pieces, when the curved, convex edge of the pie that separated the top from the side was misinterpreted by students (even by the artist!) as constituting a cut. A student other than the artist is shown counting eight *regions* as depicting eight pieces, when in fact there are only six pieces represented. Counting conventions frequently confounded students. Other students had trouble with the small triangular region in Figure 1 that resulted from the failure to have all the lines intersect in the middle.

The intended meaning of elements of students' drawings also led to confusion and lack of intersubjectivity, as shown in Figure 2. Here, the artist chose to represent the place where *cuts* were to be made, but not the resulting slices that would partition the pie. This created confusion because the top cuts are not shown as extending down through the pie, so not all agreed that this produced eight equal pieces. Some saw only six pieces.

As further evidence of the importance of intersubjectivity, representations went through continual refinement to foster greater acceptance among class members, even when they were no more correct mathematically. The solution offered in Figure 3 was an attempt to move beyond

## Visual Display and Co-Expressivity

the limits of the representations shown in Figures 1 and 2. Here, the boy on the right side of the image showed the resulting pie after three cuts were made. To help clarify what the lines on the drawing meant, he used speech and gesture as co-expressive modes; his hand was positioned vertically and moved in a vertical motion to clarify that the vertical line drawn was a slice resulting from a cut.

S: No No No . . . look, if you cut a piece of pie, see you have a line like that.

Throughout the discussion, students' talk exhibited this sort of co-expressivity—using co-incident drawings, speech, and gestures—to enhance the acceptance and interpretability of their solutions. When students failed to achieve intersubjectivity—as was evident to them by negative responses, clarification requests, or drawing moves such as adding or removing lines from others' representations—they elaborated on their depictions or made representational and conversational repairs.

Students also expanded the set of representations as the problem of achieving intersubjectivity among members of the class persisted. In one case, a student used a physical model (Figure 4) constructed out of paper, with scissor cuts used to represent slices into the pie. This model allowed the student to convey the 3D nature of the situation in a way that avoided some of the problems of the drawn representations. However, although the model was helpful for some students, it was not well designed. Each “layer” was itself made up of folded pieces of paper, and so it produced more than the eight pieces sought.

## Educational and Scientific Importance

We found ample evidence supporting Engle's (1998) claim that speakers operate with a communicative norm that their co-incident communication modes be perceived as unified and used to enhance their communicative fidelity. Neighboring speech acts were co-expressive, and the integration of multiple modes of a composite communication signal was a common response to intersubjectivity failures.

In school, students are immersed in represented worlds. Typically, we look at how well students learn to read and apply conventionalized representations to solve problems. These data provide insights into how students use invented representations along with speech to communicate their ideas about a novel problem. The data show the struggles that students have in conveying their ideas in drawn form and in interpreting the representations of others. Our student participants apparently believed that 2D drawings and explanations using speech and gesture would be sufficient to convey their solutions to their peers. They had not expected that their constructed representations would lead to divergent interpretations. Surprisingly, physical models emerged as something of a last resort and were rarely used. This finding raises questions about the nature of students' understanding and production of graphical representations, and the beliefs they acquire about formal representations. It is important for students to see drawings as formal representations in need of conventions and elaborations, rather than as self-evident. It is also vital that students understand the connections that must be established between stated problems and solutions, on the one hand, and the world that these representations are intended to portray, on the other (Palmer, 1978). This aspect of the meta-level of a discussion involving

## **Visual Display and Co-Expressivity**

representations needs to be instilled so that participants can seek a common set of understandings and collaboratively engage in productive discourse.

References

- Ainsworth, S. E. (1999). The functions of multiple representations. *Computer & Education*, 33(2/3), 131–152.
- Chandler, P., & Sweller, J. (1991). Cognitive load theory and the format of instruction. *Cognition and Instruction*, 8, 293–332.
- Engle, R. A. (1998). Not channels but composite signals: Speech, gesture, diagrams and object demonstrations are integrated in multimodal explanations. In M. A. Gernsbacher & S. J. Derry (Eds.), *Proceedings of the Twentieth Annual Conference of the Cognitive Science Society* (pp. 321–326). Mahwah, NJ: Erlbaum.
- Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science*, 11, 65–99.
- Mayer, R. E. (1999). Multimedia aids to problem solving transfer. *International Journal of Educational Research*, 31, 611–624.
- Mayer, R. E. (2003). The promise of multimedia learning: Using the same instructional design methods across different media. *Learning and Instruction*, 13, 125–139.
- Mayer, R. E., Heiser, J., & Lonn, S. (2001). Cognitive constraints on multimedia learning: When presenting more material results in less understanding. *Journal of Educational Psychology*, 93(1), 187–198.
- Mayer, R. E., & Moreno, R. (2000). A coherent effect in multimedia learning: The case of minimizing irrelevant sounds in the design of multimedia messages. *Journal of Educational Psychology*, 92, 117–125.
- McNeill, D. (1997, February). Imagery in motion event descriptions: Gestures as part of thinking-for-speaking in 3 languages. Paper presented at the Berkeley Linguistics Society, Berkeley, CA.
- Palmer, S. E. (1978). Fundamental aspects of cognitive representations. In E. Rosch & B. B. Lloyd (Eds.), *Cognition and categorization* (pp. 259–303). Mahwah, NJ: Erlbaum.
- Schegloff, E. A., Jefferson, G., & Sacks, H. (1977). The preference for self-correction in the organization of repair in conversation. *Language*, 53, 361–382.
- Seedhouse, P. (2004). *The interactional architecture of the language classroom: A conversation analysis perspective*. Malden, MA: Blackwell Publishing.
- Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, 12, 257–285.
- Tversky, B. (2001). Spatial schemas in depictions. In M. Gattis (Ed.), *Spatial schemas and abstract thought* (pp. 79–111). Cambridge, MA: MIT Press.

## Visual Display and Co-Expressivity

*Figure 1.* A student is counting the regions of the drawing rather than the pieces of pie that are supposed to be represented by the 2D drawing.

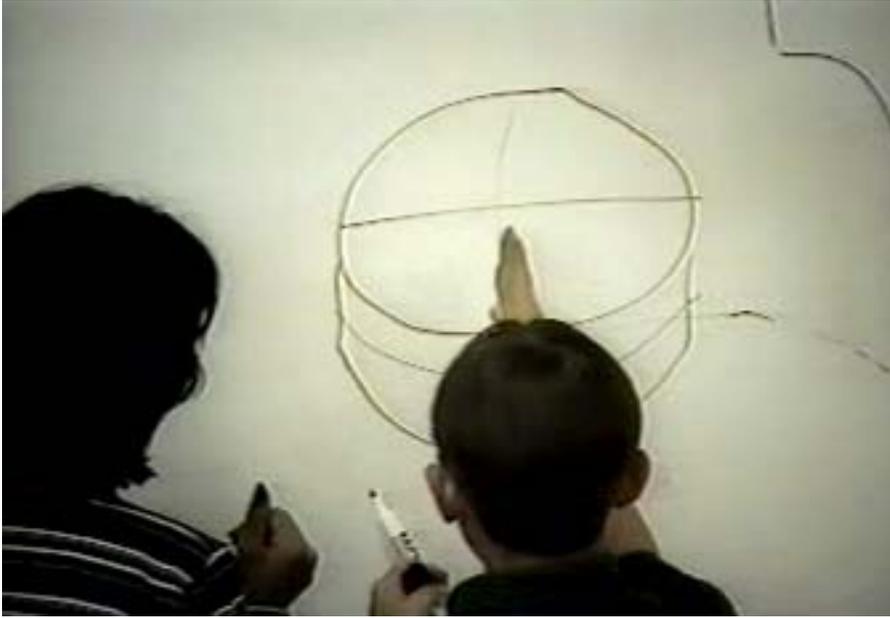


*Figure 2.* This solution representation shows the location of cuts, but not the slices and pieces that result from the cuts. Some interpreted this as showing only six pieces.



## Visual Display and Co-Expressivity

*Figure 3.* This refined solution shows the resulting slices, along with a vertical slicing hand gesture to convey the meaning of the vertical line.



*Figure 4.* A student shows a physical (paper) model that yields eight pieces after being cut only three times.

