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Mitchell J. Nathan

When Dr. Jeffrey Wigand, the protagonist of the film *The Insider* (directed by Michael Mann; written by Eric Roth; 1999), blows the whistle on nicotine research, this leading biochemistry researcher at a highly successful cigarette company leaves the corporate world and goes on to teach high school chemistry. To no one's surprise, he is a brilliant, if understated, chemistry teacher. He keeps his students sitting on the edge of their seats, completely engaged with his lessons. Viewers are expected, I suppose, to attribute Dr. Wigand's inspirational teaching to his extensive knowledge of his field. It is natural for the audience to assume that the master scientist will also be a master teacher. His years of research as a biochemist are supposed to have more than adequately prepared him to teach chemistry to 13- to 18-year old boys and girls.

Dr. Wigand's teaching rapport may be readily accepted by the typical moviegoer. But, as educators, do we accept it too easily? As it turns out, the actual Dr. Wigand did go on to teach at the high school level—but he taught Japanese, not chemistry. This little change is not merely about Hollywood retelling a story to make it more marketable. I contend that it reflects deep and abiding views about learning and teaching.

As an educational psychologist who studies teaching and learning, I am interested in the beliefs that those of us in education, and in society more generally, have about how people learn and how they should be taught. Currently, two beliefs have caught my attention. The first is that subject matter expertise is sufficient for success in teaching. The second is that to learn a specific content area, one needs to begin with an understanding of the formal structure and abstract principles that underlie the conceptual framework of the content area. This is what many people mean when they say we should start with "the basics." As one might expect, beliefs about teachers' subject matter expertise and about the path to learning through an understanding of formalisms are related. In this paper, I expose these beliefs and the foundation upon which they are built. I also describe how, together, these beliefs exert strong pressures on formal education that may not be for the betterment of learning and teaching for all students.

Belief #1: Subject Matter Expertise Is Sufficient for Good Teaching

In light of the introductory vignette, it seems reasonable to ask if those with greater expertise in a content area, such as mathematics, are better at predicting the problem-solving behaviors and difficulties of algebra students. This question is different than asking if expert mathematicians are better at doing mathematics. Instead, the question asks whether teachers who are more knowledgeable in mathematics understand their students better than teachers with less advanced subject matter knowledge. This is surely plausible. Experts in a wide range of fields—such as musical and athletic performance, strategic games, and medical practice—have been found to reason more accurately and more quickly, multitask better, and assimilate far more information than non-experts (e.g., Ericsson & Smith, 1991).

However, careful research into the cognitive processes of experts has shown that, despite perceptions and historical views to the contrary (e.g., de Groot, 1965; Galton, 1869; Salomon &

Perkins, 1989), experts function with the same internal limitations as non-experts (Ericsson & Chase, 1982; Ericsson, Chase, & Faloon, 1980; Frensch & Buchner, 1999). To exhibit superior performance, experts in many domains must train themselves to operate more efficiently within these limits through the use of refined perceptual processes, highly structured knowledge, deliberate practice, and strategies for limiting input (Bereiter & Scardamalia,1993; Ericsson, Krampe, & Tesch-Romer, 1993; Ericsson & Lehmann 1996).

Training of this sort typically leads to *routine expertise* that exploits the regularity of tasks and the environment (Hatano & Inagaki, 1986). Yet the highly tuned perception and knowledge of the expert can also lead to below-average performance on some tasks (Wiley, 1998) and poor recall of the expert's own actions and thought processes (Ericsson & Simon, 1993; Feldon, 2004). One particularly striking example comes from a case study of an expert in literature who was training to be a secondary school reading teacher (Holt-Reynolds, 1999). The subject matter expertise of this teacher was well established: She was a lifelong reader with straight A's in her college literature classes. In interviews, she demonstrated sophisticated literary analyses for a range of texts, using such techniques as intertextual references and parallel analyses of the writings, life, and times of the author. Yet this teacher's expertise did not translate into an understanding of how to model or instruct others in the reading process. Her own reading and analytic processes were so well developed and automated that they left no memory trace to reflect upon. She had no awareness of her own reading process—she did not even recognize reading as something that she once had *learned*—and she was unable to transform her own disciplinary knowledge into a form that novice learners could use and apply. As Holt-Reynolds described, this preservice teacher apparently imagined all students to be "replications of herself" (p. 41); she simply could not imagine someone not knowing how to read and needing to be taught. Her skills were so refined that she was unable to retrieve them, reflect on them, and use them as the basis for instruction. This limitation interfered greatly with her teacher training and contributed to a rather lackluster style of teaching, as evident from follow-up observations of her classroom teaching practices.

Thus, an alternative expectation is that experts' routine knowledge of a subject area may lead them to make inaccurate judgments of the actual performance demands of the task for non-experts (e.g., Brophy, 2001; Feldon, 2004).

I begin this inquiry by reviewing research on the influence of *content area expertise* on teachers' judgments of student performance in a specific area: algebra. Later, I will show that these studies reveal a common pattern that can be found in other areas of education, including science and language arts instruction.

First, let us examine students' problem-solving performance patterns when they are given problems that span arithmetic and beginning algebra. As shown in the examples of Table 1, arithmetic and algebra problems can look structurally similar yet differ in the location of the unknown quantity. For algebra problems, students are asked to reason about unknown quantities in relation to other quantities (the second row of Table 1). Students find these problems more difficult than arithmetic problems because students cannot directly model the situation as stated or directly apply arithmetic (e.g., Carpenter & Moser, 1983). In addition, the problems can vary in their presentation formats, as shown by the columns of Table 1. *Symbolic problems* use formal syntax and notation. *Verbal problems* use words and can further be compared. *Story problems* include a situational context, whereas *word equations* verbally describe the relations found in symbolic equations without an explicit problem context. Comparing performance on these problems allows us to isolate the impact of two important dimensions: context (story vs. word equation), as well as the differences between symbolic formats and verbal formats, independent of context (word equation vs. equation).

Table 1
A Sample of the Structurally Matched Problems Given to Educators to Elicit Their Expectations of Student Arithmetic and Algebraic Problem Difficulty, Organized by the Presentation Format (Columns) and the Position of the Unknown Value (Rows)

	Verbal problems		Symbolic problems
Presentation format →	Story	Word equation	Equation
Position of the unknown value ↓			
Result unknown (Arithmetic)	When Ted got home from his waiter job, he took the $81.90 he earned that day and subtracted the $66 he received in tips. Then he divided the remaining money by the 6 hours he worked and found his hourly wage. How much per hour does Ted make?	Starting with 81.90, if I subtract 66 and then divide by 6, I get a number. What is it?	Solve for X: $(81.90—66) / 6 = X$
Start unknown (Algebra)	When Ted got home from his waiter job, he multiplied his hourly wage by the 6 hours he worked that day. Then he added the $66 he made in tips and found he earned $81.90. How much per hour does Ted make?	Starting with some number, if I multiply it by 6 and then add 66, I get 81.90. What number did I start with?	Solve for X: $6X + 66 = 81.90$

Algebra students show superior performance (about 65% correct) solving the verbally presented story and word-equation problems through the strategic application of highly reliable, intuitive solution strategies, while at the same time struggling to solve carefully matched equations (getting about 43% correct; Koedinger & Nathan, 2004). This higher performance on verbal problems has been replicated for middle school, high school, and college students in several studies (Koedinger, Alibali, & Nathan, 1999; Nathan, Stephens, Masarik, Alibali, & Koedinger, 2002) and has proven quite reliable (also see Knuth, Alibali, McNeil, Weinberg, & Stephens, in press; Weinberg, 2004).

Given this pattern of performance, it is reasonable to ask how educators fare in their predictions of student problem-solving difficulty. Specifically, we would like to know how educators' predictions vary based on their levels of content knowledge. Recent research on teachers' beliefs can shed light on these questions.

In one study (Nathan & Koedinger, 2000b), teachers across the K–12 range were asked to predict the difficulty level of various mathematics problems for students just entering algebra-level instruction. With the teachers as with the students (see Table 1), problems were presented in verbal or symbolic formats, and verbally presented problems were presented with and without a story context. In all, 105 teachers from a single school district shared their expectations of problem difficulty. The teachers identified themselves either as mathematics teachers (middle and high school grades) or as elementary school teachers who taught all of the major subject areas, including mathematics. All of the high school teachers who participated in the study had been mathematics majors or reported equivalent training. In contrast, none of the middle grade mathematics teachers reported a college major in mathematics or the equivalent.

The high school mathematics teachers' predictions of student problem-solving difficulty were the least accurate. In contrast, the predictions made by middle school teachers who participated in the study were far closer to students' actual problem-solving performance and predicted the behavior of the majority of the students. Elementary teachers also were more accurate than high school teachers, though their rankings were only marginally predictive of student performance.

It is the differences between high school and middle school teachers that seem most relevant and most surprising. Middle and high school teachers of mathematics self-identify as mathematics teachers, and they each teach students within the algebra corridor. Yet the high school teachers have had substantially more mathematics education. How could they be worse at predicting student difficulty than their less mathematically advanced peers? One possibility is the nature of expertise. It is possible that the advanced mathematical knowledge of the high school teachers negatively affected their judgment. Nathan and Koedinger (2000a) speculated that these teachers may have exhibited *expert blind spot*, a phenomenon whereby teachers' own fluency with mathematics—their subject matter knowledge, per se—prevented them from seeing the difficulties that novice learners experienced (Nathan, Koedinger & Alibali, 2001). An alternative is that there are differences based on professional affiliation or expectations in primary or secondary education due, perhaps, to different course materials, curricular standards, or other factors.

A study of the beliefs of preservice teachers (PSTs) addressed these alternatives (Nathan & Petrosino, 2003). The PSTs were enrolled in a reform-based teacher education program at a major research university. Participants' knowledge of mathematics was rated high if they had completed calculus or above, and basic if their mathematics education had not reached precalculus. Those participants in the high-knowledge category can be thought of as "developing experts" in mathematics, and many went well beyond a first course in calculus, completing majors in fields of mathematics and the physical sciences.

Some of the PSTs with advanced mathematics knowledge were in a specialized program for mathematics and science majors and were seeking secondary licensure in mathematics or science education. The remaining PSTs with advanced mathematics knowledge were from the general population of teacher education students and, like the PSTs in the basic math knowledge group, sought elementary licensure. My colleague and I asked these budding educators to predict the relative problem-solving difficulty that students entering algebra instruction would experience if given problems similar to those in Table 1.

If those with advanced mathematics knowledge, regardless of whether they were enrolled in the secondary or elementary program, performed similarly to the high school teachers in the earlier study and provided relatively poor predictions of student performance, then there would be further evidence for expert blind spot. However, if the exposure to different curricula or teaching methods was the basis for inaccurate judgments of students, then we could expect to see poor predictions from those with advanced mathematics knowledge *only* if they were in enrolled in the secondary mathematics and science program.

The results strongly favored the expert blind spot hypothesis. Those with advanced mathematics education, regardless of program affiliation, misjudged student problem-solving difficulty. In fact, those with greater expertise in mathematics showed the same pattern as the high school teachers in other studies (Brophy, 2001; Bransford, Vye, Bateman, Brophy, & Roselli, 2004; Nathan & Koedinger, 2000a, 2000b). A similar result was found among preservice high school teachers in Belgium: they favored the use of algebraic methods for solving arithmetic and algebraic problems both for themselves and for their students, even when arithmetic methods were more straightforward (Van Dooren, Verschaffel, & Onghena, 2002). The views of the primary grade–level PSTs in that study were more adapted to the specific demands of the problem-solving tasks.

It is apparent from these findings that educators' content knowledge influences how they think about student performance and problem difficulty. At a general level, expert knowledge can impede instruction because it masks what is difficult and easy for novices (Bransford, Brown, & Cocking, 2000). However, I believe that it is possible to be even more specific about how expert content knowledge influences educators' views of learning and why their views systematically differ from the actual performance patterns of students.

Belief #2: Conceptual Development Proceeds From the Formal to the Applied

Recall that the high school teachers and PSTs with advanced mathematics expected students to solve symbolic equations more readily than story or word-equation problems (Nathan & Kedinger, 2000a; Nathan & Petrosino, 2003). Yet the actual performance of students in a number of studies has shown that they are far more likely to correctly solve verbally presented problems, largely because of their strategic use of invented solution methods (Koedinger & Nathan, 2004). To understand why educators with greater knowledge of mathematics are particularly inclined to misjudge student performance, it is important to examine the exact nature of content experts' predictions.

When asked to justify their predictions, the experts (see Nathan & Koedinger, 2000a) argued that symbolic reasoning was more basic and "pure" than verbal reasoning, and the natural way to introduce arithmetic and algebraic problem solving. They also argued that solving equations was a necessary prerequisite for algebra "applications" such as solving story problems, since one needs to first translate words into mathematical notation. In their responses to survey items about the development of algebraic reasoning, content experts were far more likely than non-experts to assert that symbolic reasoning ability necessarily precedes story problem solving and that symbolic procedures are the most effective method for solving mathematics problems (Nathan & Koedinger, 2000a; Nathan & Petrosino, 2003). Math experts were also less likely to believe that students enter the classroom with intuitive methods for reasoning about algebra-level problems, and they tended to discount the role that students' invented solution methods could play as the basis for further mathematics instruction and learning.

It is not my intention here to question the power and utility of formalisms. Formal representations such as symbolic equations, graphs and diagrams, scientific laws, and formal theories are vitally important because of the central organizing role they play in a given discipline. Formalisms support computational efficiency and can mitigate ambiguity (e.g., Tabachneck, Koedinger, & Nathan, 1995). Formalisms can also reveal the common deep structure of quantitative and qualitative relations of seemingly disparate phenomena (such as mechanical and electrical circuits) and thereby provide important conceptual bridges to support transfer, discovery, and theory building. These are properties of formalisms that serve the needs of those with competence in their field. However, the primary role formalisms play in organizing a given discipline for experts appears to have been appropriated by many educators as the model of conceptual development for one initially coming to learning in that discipline. As educators, we need to be careful to distinguish the conceptual structure as apparent to experts in a discipline, from Bruner's (1960) account of structure as that which makes salient the relations among seemingly unrelated things for the purpose of transfer. "If earlier learning is to render later learning easier, it must do so by providing a general picture in terms of which the relations between things encountered earlier and later are made as clear as possible" (p. 12). For an area like algebra, it is common to believe that algebraic equations constitute that structure and that students will naturally and necessarily reason about formal mathematical symbols before they can reason about mathematical relations and problems presented in words. Nathan and Koedinger (2000b) labeled this the *symbol precedence view* of algebraic development.

The symbol precedence view has a great deal of intuitive appeal, is evident among popular algebra textbooks (Nathan, Long & Alibali, 2002), and leads to a natural justification for educating novices by teaching formalisms first. However, the data on student performance do not support this view. Koedinger and colleagues (Koedinger et al., 1999; Koedinger & Nathan, 2004) found that high school and college students, despite formal instruction in pre-algebra and algebra, often did not know what to do with equations; they frequently gave no response to these problems, indicating a basic failure to comprehend the meaning of the notations. When students did try to apply symbolic solution methods directly, their attempts were highly error-prone.

This overall finding is not restricted to algebra. A multiyear study of over 400 undergraduates showed that beginning physics students were more likely to correctly answer questions involving Newton's Third Law when those questions were presented verbally rather than using vector diagrams (Meltzer, 2005).

In light of these findings, it may not be so surprising that the pattern of student algebra problem-solving performance is, in fact, more consistent with a *verbal precedence view* of development: Several studies (e.g., Koedinger & Nathan, 2004; Nathan et al., 2002) have now shown that verbally presented “application” problems are far more accessible, and they are solved correctly more often, because students seem to comprehend them better and are therefore able to make use of *intuitive* solution methods that circumvent the need for formal notation and equations. These intuitive methods seem to have been invented on demand in many cases, drawing on the meaning of the problems formed by the students and students’ knowledge of the behavior of quantities.

Students were also more likely to solve symbolic problems correctly if they also correctly solved verbal problems (e.g., Nathan & Koedinger, 2000b). It was extremely rare to see students who could solve the arithmetic and algebraic equations but not the verbal problems (only 1 in 76 students; Nathan & Koedinger, 2000b). This finding suggests that the development of students’ symbolic reasoning may in fact depend on and draw on their verbal reasoning skills, a notion that has support from research in a variety of other areas of intellectual development (e.g., Case, 1991; Case & Okamoto, 2000; Kalchman, Moss, & Case, 2001).

One explanation for students’ poor understanding of algebraic formalisms is that students do not achieve a *grounded* understanding of the meaning of the symbolic equations in terms of other concepts they already understood (Koedinger & Nathan, 2004). Algebra is an arbitrary system of representation that obtains its power through its abstract nature. In this way, algebraic symbols are “second-order” descriptions of actual objects and events in the world (Laurillard, 2002). The computational properties of algebra come about because of the rules that govern the relations and transformations of this system of notation. Yet, when rules and symbols are merely understood through self-reference to other second-order descriptions in the form of abstract rules and symbols, it leads to an ungrounded form of understanding. Meaning, I would argue, comes ultimately from reference to nonsymbolic entities such as perceptions and experiences from the world (Glenberg, 1997, 1999; Harnad, 1990; Searle, 1980). Indeed, some of the most profound advances in mathematics—such as the creation of negative numbers, logic, and infinity—have been shown to have been achieved by prominent mathematicians who made the appropriate links to the grounded behavior of objects and events in the everyday world (Lakoff & Johnson, 1999; Lakoff & Nunez, 2000). And it is through grounded relationships that these formal entities ultimately attain their meaning and utility.

The symbol precedence view in mathematics education is but one of many views of conceptual development that give formalisms a privileged status in education. In science education, formal symbols and laws are seen as primary, and their understanding is considered crucial for students’ later success with physics word problems and for understanding technology and other scientific applications (e.g., Bloomfield, 1998; Cajas, 1998, 2001). In language arts, teachers who are expert in literature but have never been in a teacher education program often see formal methods of textual analysis and the rules of grammar as prerequisites to students’ reading and creative writing performance (e.g., Grossman, 1990). In my own field of educational psychology, I see how formal theories and domain-general principles of learning and development are taken as prerequisites, to be understood before teacher education students can apply them to specific areas of instruction. In a related fashion, the practical skills needed to perform technical and service trades—such as carpentry, plumbing, and hairstyling—are generally held in lower regard within the public education system than the knowledge associated with an academically oriented education (Rose, 2004). Across a wide range of areas, knowledge of formalisms is privileged over practical and applied knowledge.

The privileged view of formalisms and theory over applied knowledge is historically rooted. That is, biases about the elevated status of formal and theoretical knowledge as compared with applied knowledge seem to be based not on a scientific understanding of the manner in which learning occurs or of the relative intellectual demands of formal and practical skills. Rather, these judgments—prevalent in education and in society as a whole—seem to be based largely on the character of those demands. As Rose (2004) states, "the more applied and materialized the mathematics is, the less intellectually substantial it is" (p. 98).

The split between the practical work of the "hand" and the intellectual work of the "brain" is woven into the very fabric of people's thinking about science in the modern age. Vannevar Bush, the first director of the U.S. Office of Scientific Research and Development, wrote that "basic research"—that is, "research performed without thought of practical ends"—is preeminent and the driving force behind the major advances in science, whereas "applied research drives out the pure" (Stokes, 1997, p. 3). This fervent bias in favor of formal reasoning and theoretical research was evident as early as the Classic Hellenic Era, when scientific inquiry was considered to reach its highest form when it was purely for the pursuit of knowledge (the "philosophical arts") and offered no practical outcomes (as in the "manual arts"; Plato, *Republic VII*).

The conceptions of training for those who specifically choose to pursue fields of applied science and mathematics are also steeped in this view. The Nobel laureate Herbert Simon (1969/1996) observed that "Engineering schools gradually became schools of physics and mathematics; medical schools became schools of biological science, business schools became schools of finite mathematics" (p. 111). As Cajas (1998) noted, this is still true 30 years later:

> The way in which future technologists (e.g., engineers or medical doctors) are generally prepared is the following: Students first take science classes with the assumption that such classes can be applied to specific technological problems (e.g., engineering problems, medical problems). The justification of taking science classes (physics for example in the case of engineers or physiology in the case of medicine) is that these classes are the bases of their future professional work (p. 5).

I have even encountered this view lurking in a book on the hobby of kite making. The book (Hosking, 1992) is full of practical aspects of kite design, with many traceable designs, kite-flying techniques, and so on. Yet the author felt compelled to provide a scientific primer on lift and air pressure in the opening chapter, complete with vectors, diagrams with angles of attack, and illustrations of airflow. The role of this formal knowledge in advancing kite design must be acknowledged, surely. Many expert kite makers and fliers have a strong formal and intuitive understanding of the science. But its utility is doubtful for those who are just learning to build and fly kites. It, too, reveals beliefs the author has about how a book on kites can be legitimized by its ties to formalisms in physics.

Implications

Well-developed content knowledge is, of course, vital for teachers, and it is legitimate to expect it from teachers at all grade levels. Yet, with advanced knowledge come certain biases about one's field and about how others will learn the knowledge and participation norms for that field. The formalism-first view of conceptual development, although lacking strong empirical support, is widespread among educators and throughout society. In this last section, I explore some of the implications of these beliefs for education, particularly their effect on classroom and curricular practices and on education policy.

Classroom and Curricular Implications

Teachers' beliefs about the developmental processes of their students serve as the bases for their judgments about classroom assignments, selection of curriculum materials, and even the evaluation of students' thinking and progress (Grossman, 1990). Misconceptions about the developmental process can lead teachers, curriculum designers, and education leaders toward non-optimal, or even incorrect, conclusions about the progress of a child. For example, a teacher may withhold algebra story problems from an algebra student who is struggling with equations based on beliefs about the relative difficulty of these tasks. As a result, the teacher may never see the informal reasoning approaches available to that child that could form the basis of future algebra understanding. Alternatively, a teacher may take the correct or expected answer as a direct indicator that the student has learned the taught method, unaware that the student has actually used another method and thus achieved only a "veneer of accomplishment" (Lave, Smith, & Butler, 1988; Hennessy, McCormick, & Murphy, 1993).

Education Policy Implications

One of the greatest problems stemming from the formalism-first view is that it undermines some of the central tenets of public education, particularly for those students who focus on vocational and technical education. Public education is entrusted with providing equal access to excellent educational opportunities. But, in practice, schools restrict that access for students who pursue a technically oriented education instead of college. Career and technical education courses, with their focus on less valued applied skills, are often completely devoid of the theoretical and formal content that educators consider necessary to support later generalization and abstraction. Rose (2004) calls this the "fundamental paradox of vocational education" and argues that the lack of attention to theory and generalization in vocational education classes withholds essential knowledge and perpetuates stereotypes of who is capable of abstract thought and worthy of the tremendous resources of the educational system to foster upward economic and social mobility.

Belief in the centrality of discipline-specific formalisms has also shaped recent policy and practices of teacher licensure. By downplaying the value of practical knowledge of teaching that preservice teachers acquire through their teacher education programs and privileging the development of content knowledge, the education community has created a double standard for teacher licensure. Federally funded programs like Transition to Teaching provide a streamlined licensure process for content experts to recruit mid-career professionals and recent college graduates to classrooms. However, teacher education graduates must go through full licensure and also pass standardized tests of subject matter knowledge and pedagogy in the areas in which they will teach. These policies can be justified through an appeal to the power of formal content knowledge in much the same way that algebra teachers justify their predictions of student problem-solving difficulty and that we justify our acceptance of the Hollywood story of biochemist turned dazzling high school chemistry teacher. Ironically, the most likely cause of teacher ineffectiveness is a deficiency in pedagogical content knowledge, whereas deficiencies in content knowledge are the least likely cause (Torff, 2005). Clearly, these deep-seated views must be called into question when they promote inaccurate views of learning.

I personally agree with the essential importance of well-developed subject matter knowledge: Teachers need strong content knowledge to understand their curricula and their students. However, "subject matter expertise across disciplines can, if unchecked, lead teachers to be blind to certain developmental needs of novice learners" (Nathan & Petrosino, 2003, p. 921) and to favor instructional approaches that build from the formalisms central to that discipline. This may be surprising, even counterintuitive. But now that I know to look, I see this pattern in many surprising places, even in my own field of educational psychology. We, as members of the educational community, need to look deeper at the relationship between content knowledge and teaching, and we need to acknowledge that our beliefs about learning and education should be subject to the same scrutiny we expect from any scientific endeavor that has such a profound influence on the youth of today and the education of future generations.

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