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Can Achievement Peer Effect Estimates Inform Policy? A View from Inside the Black Box*

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Abstract

Empirical studies of peer effects rely on the assumption that peer spillovers can be measured through observables. However, in the education context, many theories of peer spillovers center around unobservables, such as ability, effort or motivation. I show that when peer effects arise from unobservables, the typical empirical specifications will not measure peer effects accurately, which may help explain differences in the magnitude and even sign of peer effects estimates across studies. I further show that under reasonable assumptions these estimates cannot be applied to determine the effects of regrouping students, a central motivation of the literature.

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1 Introduction

Understanding peer effects is important for a large number of educational policies that either directly or indirectly alter the grouping of students such as bussing for racial integration, ability grouping, the introduction of charter schools and private school vouchers. While empirical studies of peer effects in achievement production abound, they provide mixed evidence regarding the magnitude and even sign of these effects, severely limiting the potential to inform policy.[1] The empirical literature is generally based on the assumption that peer effects derive through behaviors that are observed to the researcher.[2] However, unlike much of the broader social interactions, in the education context the underlying theories of peer effects center around behaviors that are unobserved to the researcher. In this paper, I consider how this unobservability changes the interpretation of the peer spillovers typically estimated in empirical specifications (e.g., Hanushek et al., 2003). This provides new insight into why estimated spillovers from peer characteristics in achievement production are ambiguous in sign and the potential for these models to inform policies related to regrouping students.

The typical empirical specification, which I term the *statistical model*, assumes achievement is potentially a function of both peer characteristics (such as race, sex, socioeconomic status) and peer achievement. Based on the pioneering work of Manski (1993), much of the literature has focused on the challenges associated with identifying the parameters of this model. The central contribution of this paper is to show that even if the statistical model is identified, the estimated peer effects may be difficult to interpret and apply to policy. The argument centers around the observation that it is not peer achievement per sé that affects a student's own achievement. Rather, the fundamental intuition for including peer achievement in the peer effects regression is that something unobservable about the peers,

[1] For instance, see Schofield (1995) for a review of the mixed evidence regarding the effect of desegregation.

[2] See Brock and Durlauf (2001*b*) for an overview.

such as their ability, motivation or behavior, matters, and peer achievement can proxy for these unobservable traits.

Recognizing peer achievement as a proxy for unobservables, I show that after conditioning on peer achievement (contemporaneous or lagged), the estimated effects of peer characteristics may be biased toward 0 or even counterintuitive in sign. For example, suppose that having better educated parents is positively correlated with achievement. Then, conditional on a given level of peer achievement, the higher the parental education of peers the lower the peer ability or effort. It is the fact that peer parental education partially proxies for the unobservable input of peers that drives the estimated effect of peer parental education downward and even negative, depending on the relative magnitude of the unobservable peer effect.

Not only does this help explain some of the mixed evidence in the literature, but it suggests considerable care needs to be taken in determining how to apply empirical estimates to policy. Most directly, the peer spillovers that are estimated based on measures of "ability" (such as standardized exams) that are observed by the researcher may not be the appropriate parameters to apply to determine regrouping when the school or district is using other measures of "ability" (such as GPA or IQ scores).

I further show that the implications for applying empirical estimates to the questions of regrouping students can be quite different depending on the source of unobservables. I distinguish between two different types of unobservables that may affect achievement–(1) peer characteristics, such as ability and (2) peer behaviors, such as effort or motivation. Ability is assumed to be predetermined, whereas effort may change based on peer or teacher inputs.

The distinction between ability and effort is important since any regrouping of students implicitly involves a reassignment to teachers. If peer effort matters and responds to changes in teachers or peers,[3] being assigned a better teacher will have both a direct effect on a student's behavior and an indirect effect deriving through the improvement in her peers' behaviors. Both of these raise classroom achievement. The teacher effect then multiplies as the student's behavior affects her peers' behavior and vice versa (a *social multiplier effect*). If there is matching between teachers and students in the data, as is generally the case with observational data, estimates of the social multiplier effect (or the spillovers from peer achievement) are generally necessary (under reasonable assumptions) to determine the effects of regrouping. This is a surprising result, as it is often assumed that estimates of the spillovers from peer achievement are not important when grouping is based on observable peer characteristics. Furthermore, this is not the case if spillovers derive only through unobserved ability which is neither affected by the new teachers or peers implicit in the regrouping.

While throughout most of the paper I assume the conditions for identification of the statistical model, I also discuss how implications for identification of the spillovers from peer achievement differ across types of unobservables. I show why this distinction may matter for identification. The key distinction is that effort spillovers imply a simultaneity problem (i.e., Manski (1993)'s *reflection problem*) whereas ability spillovers do not.

My paper is not the first to observe that peer achievement really proxies for unobservable peer inputs. Hanushek et al. (2003) and others acknowledge unobservable ability as the underlying rationale for controlling for peer achievement in the peer effects regression. To the best of my knowledge, Arcidiacono et al. (2009) is the only paper to make this explicit in their approach to estimating peer effects, and Burke and Sass (2006) use their approach to estimate peer effects in Florida schools. What I contribute to the existing literature is to develop the implications of the peer effects deriving from unobservables for the interpretation of spillovers from observable peer characteristics. I also draw important distinctions between different types of unobservables and discuss the limitations in applying the typical empirical

[3]See Bishop et al. (2003),Kinsler (2006), among others.

specification to policy.

I begin in Section 2 by providing some background on the statistical model that is typically estimated in the literature and then write down a theoretical model that makes explicit the argument that peer achievement spillovers derive through unobservables. I begin with the simplest case where the unobservable is exogenous ability, as described in Section 2.1, and build to the endogenous effort case in Section 2.2. I develop implications for the capacity of statistical models to inform regrouping policies in Section 3, contrasting grouping policies based on observable and unobservable peer characteristics. While throughout most of the paper I assume a simple setting where peer effects are homogeneous across student types, I further show how moving to the heterogeneous spillovers frameworks raises additional concerns about the capacity of statistical models to inform grouping policies. In Section 4, I discuss identification of endogenous peer effects and how it varies based on the underlying type of unobservable peer effect. I further briefly consider the implications when we incorporate dynamics in the model in Section 5. Section 6 concludes.

2 Interpreting Contextual Effects

Let Y_i denote achievement on a standardized exam for a student i. For expositional purposes, I consider a case where the peer group consists of two students ($i = \{1, 2\}$), though the arguments below can easily be extended to multiple students in the peer group. Observed individual characteristics, which often include parental education, race, sex, and some measure of income are denoted X_i. I ignore dependence on shared observable classroom inputs, such as teacher experience or expenditure, as these are not central to the analysis, and instead include only shared classroom inputs μ, such as teacher quality, which are unobservable to the researcher.

The education literature generally takes as a starting point an achievement production function with peer spillovers of the following form:

$$Y_i = X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j\tilde{\gamma}_y + \mu\gamma_\mu + \xi_i,\ i \in \{1,2\},\ j \neq i, \tag{2.1}$$

where peer spillovers derive both through peer characteristics X_j (exogenous or contextual effects) and peer achievement Y_j (the "endogenous" effect). I use the $\tilde{\gamma}$ to distinguish peer effect parameters. Throughout the paper, I refer to this as the *statistical model.*

In other branches of the social interactions literature where similar specifications are estimated, the potential importance of endogenous effects for determining behavior is relatively self evident. Consider for instance the decision of a teenager to smoke or drink alcohol. Few would argue that the tendency toward such behaviors is unaffected by peer pressure, i.e., by having peers that engage in these behaviors. Yet, the role played by endogenous effects in achievement production is less clear, leading to considerable confusion regarding peer spillovers in the achievement production context. Annual standardized exams are often the outcome of interest, and, in the absence of cheating, are not a group effort. Thus, peer achievement per sé may not affect a student's achievement, in the same sense it does in the teenage smoking example.

Yet, despite this observation, some measure of peer achievement is often included among the potential peer effects in the achievement production function and is frequently even the input of interest. In reality, peer achievement may signal something about peers that affects achievement production. There are several theories underlying the inclusion of peer achievement as an input to production. I distinguish between two–one that treats peer achievement as capturing unobserved characteristics of the student (such as ability) and another as capturing an unobserved action or behavior of the student (such as effort).

In this section, I focus particularly on the implications of treating the "endogenous" peer effect as arising through unobservables for the interpretation of the contextual peer effects,

$\tilde{\gamma}_x$. Throughout the section I make the assumption that the econometrician chooses to use achievement to proxy for the unobservable. This contrasts with the question of whether peer achievement satisfies the conditions needed to make it a *good* proxy. The fact that it is not a good proxy in fact underlies some of the identification problems and the counterintuitive interpretation of contextual effects described below.

I begin by explicitly modeling the ability spillover case in Section 2.1, treating students as passive inputs to the achievement process. While I believe that this model misses the potentially more important behavioral effects of peers, it provides the clearest exposition of the implications for interpreting contextual peer effects in the context where peer achievement is proxying for an unobservable. I then turn to the peer effort case in Section 2.2. While the implications for contextual peer effects are similar, the contrast becomes important for policy and identification, as discussed further below.

2.1 Exogenous Unobservable–Ability

I begin with the simplest setting where the unobservable A_i, say ability, is exogenous or predetermined. Suppose the structural achievement production function takes the form

$$Y_i = X_i\alpha_x + X_j\tilde{\alpha}_x + A_i\alpha_a + A_j\tilde{\alpha}_a + \mu + \epsilon_i, \tag{2.2}$$

for each $i \in \{1, 2\}$, $i \neq j$. In this production function, peer j affects i's achievement through observable characteristics (X_j) and unobservable ability (A_j). Suppose we want to know the effect of peer ability. Assume that achievement is increasing in ability, i.e., $\alpha_a > 0$. Then, peer achievement can serve as a proxy for peer ability.

Solving for peer ability as a function of peer achievement and other inputs and substituting for peer ability in equation (2.2) yields

$$\begin{aligned}Y_i =&X_i\left(\alpha_x-\tilde{\alpha}_x\frac{\tilde{\alpha}_a}{\alpha_a}\right)+X_j\left(\tilde{\alpha}_x-\alpha_x\frac{\tilde{\alpha}_a}{\alpha_a}\right)+Y_j\frac{\tilde{\alpha}_a}{\alpha_a}\\&+A_i(\alpha_a-\frac{\tilde{\alpha}_a^2}{\alpha_a})+\mu(1-\frac{\tilde{\alpha}_a}{\alpha_a})+\epsilon_i-\epsilon_j\frac{\tilde{\alpha}_a}{\alpha_a}\\ \equiv&X_i\gamma_x+X_j\tilde{\gamma}_x+Y_j\tilde{\gamma}_y+A_i\gamma_a+\mu\gamma_\mu+\epsilon_i-\epsilon_j\tilde{\gamma}_y.\end{aligned} \tag{2.3}$$

The question of interest is how to interpret the contextual effects parameter, $\tilde{\gamma}_x$. Without loss of generality, assume that the covariates are constructed such that $\alpha_x \geq 0$.

Table 1: Ability Model Parameter Assumptions

	Structural Parameters	Statistical Model Parameters
(1)	$\alpha_x \geq 0$	without loss of generality
(2)	$\alpha_a > 0$	(2) $\Rightarrow$ Y proxy for ability
(3)	$\alpha_a \geq \tilde{\alpha}_a > 0$	(2), (3) $\Rightarrow$ $\tilde{\gamma}_y \in (0,1]$
		(2), (3)$\Rightarrow$ $\tilde{\gamma}_\mu \in [0,1)$
(4)	$\alpha_x \geq \tilde{\alpha}_x$	(1)-(4) $\Rightarrow$ $\gamma_x \geq 0$
		(1)-(4) $\Rightarrow$ $\tilde{\gamma}_x \lesseqqgtr 0$
(5)	$\frac{\tilde{\alpha}_x}{\alpha_x} > \frac{\tilde{\alpha}_a}{\alpha_a}$	(1), (2), (5) $\Rightarrow$ $\tilde{\gamma}_x$ same sign as $\tilde{\alpha}_x$

To aid in interpretation, Table 1 details assumptions on the parameters of the structural model (the α's) and the implications for the parameters of the statistical model (the γ's). As this is the interesting case to consider, I assume that peer ability spillovers exist, and, consistent with intuition, they are positive, i.e., $\tilde{\alpha}_a > 0$. Imposing also the intuitive assumption that the marginal effect of peer j's ability does not exceed the marginal effect of i's own ability, i.e., $\alpha_a \geq \tilde{\alpha}_a$, the endogenous effect parameter $\tilde{\gamma}_y \in (0,1]$.

Assuming that the direct effect of i's observable characteristics is at least as large as the effect of peer j's characteristics, i.e., $\alpha_x \geq \tilde{\alpha}_x$, it follows that $\gamma_x \geq 0$. In other words, the estimated effect of i's characteristics in the empirical specification will have the right sign. It is however biased toward 0 if $\tilde{\alpha}_x > 0$. A similar intuition holds for estimates of the effect of shared classroom inputs μ. However, under similarly reasonable assumptions, the sign of the effect of peer characteristics is ambiguous.

To illustrate, start by assuming that there is no direct effect of peer characteristics on achievement, i.e., $\tilde{\alpha}_x = 0$. Then, $\tilde{\gamma}_x = -\alpha_x \tilde{\gamma}_y$. Assuming that $\alpha_x \neq 0$, so that the individual characteristic matters for achievement, the contextual peer effect still enters the statistical model as a proxy for peer ability but negatively, taking the opposite sign of the individual effect α_x, counter to the usual intuition regarding the sign of contextual effects. Intuitively, this occurs because conditional on a given level of peer achievement, a higher level of peer characteristics actually predicts a lower level of unobserved peer ability.

If there are direct spillovers from peer characteristics in achievement production the sign of the contextual effect in the statistical model is ambiguous because of the countervailing influences of the indirect effect of peer characteristics as proxying for unobserved peer ability and the direct effect of peer characteristics in achievement production. Intuitively, this suggests that the stronger the spillovers from peer ability, the stronger the direct effect of the individual characteristic, and the weaker the direct effect of peer characteristics, the more likely $\tilde{\gamma}_x$ is to take a "counterintuitive" sign.

A necessary condition for the statistical parameter, $\tilde{\gamma}_x$, to take the same sign as the direct effect of peer characteristics ($\tilde{\alpha}_x$) is that $\tilde{\alpha}_x > \tilde{\gamma}_y \alpha_x$ or $\frac{\tilde{\alpha}_x}{\alpha_x} > \frac{\tilde{\alpha}_a}{\alpha_a}$. In words, it must be that the relative effect of the observable characteristic of peer j to the direct effect of the observable characteristics for i must exceed the same ratio for the unobservables.

A particular example of a contextual effect may help clarify the above results. Consider the effect of parental education. The literature generally supports the finding that students with better-educated parents perform better in school on average. This could follow if for instance better-educated parents value education more and are more able or more willing to spend time teaching their child outside of the classroom, helping on homework assignments, and facilitating other activities conducive to achievement.

But, the rationale for an effect of the parental education of peers on achievement is less clear. At one level, parents are not in the classroom and therefore cannot directly affect achievement production. Yet, peer parental education could potentially affect the productivity of the teacher, for instance, if better-educated parents spend more time monitoring and this has positive spillovers for the classroom. However, conditional on a given set of teacher inputs, peer parental education may have less of a direct effect on the productivity of the classroom. Therefore, this suggests a setting where conditioning on teacher productivity we could find a negative effect of peer parental education on achievement, simply because the indirect effect of peer parental education as a proxy for peer ability dominates the direct effect.

2.2 Endogenous Unobservable–Effort

While the above model provides the basic intuition for why the contextual effect that is estimated in the statistical model may be difficult to interpret, the assumption of an exogenous unobservable is a restrictive one. Suppose we allow for another type of unobservable, effort or e_i, that is endogenous. The structural achievement production function is now

$$Y_i = X_i\alpha_x + X_j\tilde{\alpha}_x + A_i\alpha_a + A_j\tilde{\alpha}_a + e_i\alpha_e + e_j\tilde{\alpha}_e + \mu + \epsilon_i. \tag{2.4}$$

In addition to the contextual effects deriving through observable peer characteristics and unobservable peer ability, this specification permits a third type of peer effect, deriving through peer effort. As this is a behavioral choice of the student, I term this the *endogenous* effect to contrast with case in the previous setting where the unobservable ability was more like an unobservable contextual effect. Similarly to above, I assume that $\alpha_x \geq 0$ without loss of generality.

Normally, the literature does not distinguish between unobserved effort and ability of students, yet both have a place in theories of peer spillovers. The tracking literature assumes that being grouped with higher ability peers benefits students. This could occur through many channels. For instance, higher ability students may ask better questions, from which their classmates benefit. Or, it could be that higher ability students can help teach their classmates in group work settings.[4]

Previous studies also support an effect of peer effort on achievement. For instance, Lazear (2001)'s model of peer influence predicts that the disruptive behavior of a student imposes negative externalities on other students in the classroom. Both Figlio (2007) and Kinsler (2006) present empirical evidence that disruptive peers negatively affect achievement. Equally plausible is the potential positive externality of being grouped with more engaged students.

Another important implication of this production specification is that by allowing a student's effort to affect achievement production, students are explicitly treated as active rather than passive inputs to the achievement process. In this case, student incentives matter. While student incentives have received considerable attention elsewhere in the achievement literature (e.g. Bishop and Woessmann, 2004; Bishop, 2006; Costrell, 1994), for the most part students are treated as passive inputs in achievement production functions. Treating students as decision makers introduces a natural role for peers in shaping student incentives, by setting norms of conduct and providing social pressures against or in favor of achievement.

To understand the student's incentives, it is useful to write down the student's utility in a form similar to other social interaction models estimated in the literature (e.g., Brock and Durlauf, 2001*b*). Suppose the student derives utility from achievement, and there is a cost

[4] In fact, it is likely that the two effects interact in the sense that high ability students are unlikely to provide positive spillovers if they do not exert effort. The simple, additively separable setting is maintained here to generate the linear-in-means statistical model.

to exerting effort that depends on the effort of her peers, i.e.,

$$U_i = u_i(Y_i) - c_i(e_i, e_j),$$

where $\partial u_i(\cdot)/\partial Y_i > 0$, $\partial c_i(\cdot)/\partial e_i > 0$ and $\partial^2 c_i(\cdot)/\partial e_i \partial e_j \neq 0$. If $\partial^2 c_i(\cdot)/\partial e_i \partial e_j \geq 0$, this is a conformity type effect (as discussed by Brock and Durlauf (2001*a*) and others in the broader social interactions context), where a student seeks to conform to the effort of her peers. In principle, the effect could go in the opposite direction, if having better peers leads to discouragement. The i subscripts permit utility maximizing behavior to vary by individual and peer characteristics.

By modeling the students' incentives, I introduce two important new channels for peer effects in the achievement context. To provide a familiar example in the context of achievement, the cost of working hard in a class of non-hard-working peers is likely to be much larger because the student risks standing out as a "nerd" or "teacher's pet."[5] Furthermore, peer characteristics can enter the utility function, for instance, if having peers with better-educated parents leads students to value achievement more.

I assume that the observed achievement derives from students' utility-maximizing effort. Suppose that utility takes the form

$$U_i = \beta_{yi} Y_i - \frac{\beta_e}{2} e_i^2 + \tilde{\beta}_e e_i e_j,$$

where $\beta_{yi} \geq 0$, $\beta_e \geq 0$. This simplification is useful because it obtains an equivalent of the statistical model in equation (2.1) that is linear-in-parameters in achievement and peer characteristics as an equilibrium outcome of students' utility maximizing efforts.[6] The cost

[5]For instance, see Bishop et al. (2003). In a competitive environment, students may value achievement mostly as it relates to their peers' achievement, producing a similar style peer effect in determining optimal behavior.

[6]Cooley (2009) describes the derivation of an achievement best response from an equilib-

of effort in this model takes the form that Brock and Durlauf (2001*a*) term the *proportional spillovers* case, permitting complementarity in effort if $\tilde{\beta}_e \geq 0$ and discouragement type effects otherwise. Allowing the marginal utility of achievement to vary across individuals permits variation in utility-maximizing effort.[7] Note that this type of heterogeneity is new to the literature on social interactions, where heterogeneity is generally driven by terms in the residual. As I demonstrate below, it also proves to be an important generalization for identification.

Given that students simultaneously choose effort to maximize expected utility, a student i's best response to any given level of peer effort is

$$e_i^{BR} = \frac{\beta_{yi}\alpha_e}{\beta_e} + \frac{\tilde{\beta}_e}{\beta_e} e_j.$$

Utility-maximizing effort is a function of the marginal utility of effort relative to the cost and is increasing in the average effort of peers as a result of the conformity effect.[8] In previous work, Cooley (2009), I show the informational assumptions and other conditions needed for a Nash equilibrium to exist in a more general setting. In the present context, the equilibrium described below is consistent with various types of informational assumptions given the additive separability in the residual and other classroom inputs.

Assuming $\alpha_e > 0$ so that achievement is monotonically increasing in effort, the effort best response maps into an achievement best response, which is observable to the econometrician.

rium model of effort that is similar in nature to the statistical model described in 2.1, but with a much more general functional form.

[7] Note that as it is the marginal rate of substitution that matters, restricting β_e to be homogeneous is without loss of generality. However, in a model of endogenous reference group formation it seems likely that $\tilde{\beta}_e$ might also be individual-specific, i.e., where individuals place more weight on the actions of peers more "like" themselves. While this has interesting implications, it is beyond the scope of the present paper.

[8] Note that allowing for effort and peer effort complementarities in the achievement production function would suggest that the best response is increasing in average peer effort even in the absence of the conformity effect.

Solving for effort as a function of achievement using the production function in (2.4) yields

$$e_i = \frac{1}{\alpha_e^2 - \tilde{\alpha}_e^2}(Y_i\alpha_e - Y_j\tilde{\alpha}_e - X_i(\alpha_x\alpha_e - \tilde{\alpha}_x\tilde{\alpha}_e) - X_j(\tilde{\alpha}_x\alpha_e - \alpha_x\tilde{\alpha}_e) - A_i(\alpha_a\alpha_e - \tilde{\alpha}_a\tilde{\alpha}_e)$$
$$- A_j(\tilde{\alpha}_a\alpha_e - \alpha_a\tilde{\alpha}_e) - \mu(\alpha_e - \tilde{\alpha}_e) - \epsilon_i\alpha_e + \epsilon_j\tilde{\alpha}_e)).$$

Plugging i's effort best response into the achievement function and proxying for peer effort through peer achievement and other variables as above results in the achievement best response, i.e.,

$$\begin{aligned}
Y_i^{BR} =& \frac{\beta_{yi}\alpha_e(\alpha_e^2 - \tilde{\alpha}_e^2)}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e} + X_i\left(\alpha_x - \frac{\tilde{\beta}_e\alpha_e + \beta_e\tilde{\alpha}_e}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}\tilde{\alpha}_x\right) + X_j\left(\tilde{\alpha}_x - \frac{\tilde{\beta}_e\alpha_e + \beta_e\tilde{\alpha}_e}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}\alpha_x\right) \\
&+ Y_j\left(\frac{\tilde{\beta}_e\alpha_e + \beta_e\tilde{\alpha}_e}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}\right) + A_i\left(\alpha_a - \frac{\tilde{\beta}_e\alpha_e + \beta_e\tilde{\alpha}_e}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}\tilde{\alpha}_a\right) \\
&+ A_j\left(\tilde{\alpha}_a - \frac{\tilde{\beta}_e\alpha_e + \beta_e\tilde{\alpha}_e}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}\alpha_a\right) + \mu\left(1 - \frac{\tilde{\beta}_e\alpha_e + \beta_e\tilde{\alpha}_e}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}\right) + \epsilon_i - \epsilon_j\left(\frac{\tilde{\beta}_e\alpha_e + \beta_e\tilde{\alpha}_e}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}\right), \\
\equiv& \frac{\beta_{yi}\alpha_e(\alpha_e^2 - \tilde{\alpha}_e^2)}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e} + X_i(\alpha_x - \tilde{\gamma}_y\tilde{\alpha}_x) + X_j(\tilde{\alpha}_x - \tilde{\gamma}_y\alpha_x) + Y_j\tilde{\gamma}_y \\
&+ A_i(\alpha_a - \tilde{\gamma}_y\tilde{\alpha}_a) + A_j(\tilde{\alpha}_a - \tilde{\gamma}_y\alpha_a) + \mu(1 - \tilde{\gamma}_y) + \epsilon_i - \epsilon_j\tilde{\gamma}_y, \\
\equiv& \gamma_{0i} + X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j\tilde{\gamma}_y + A_i\gamma_a + A_j\tilde{\gamma}_a + \mu\gamma_\mu + \epsilon_i - \epsilon_j\tilde{\gamma}_y.
\end{aligned}$$

Given that the achievement best response is linear-in-parameters, it can be shown that a unique Nash equilibrium exists to this game, (Y_1^*, Y_2^*).[9] The observed equilibrium achieve-

[9] While I assume that students are able to perfectly predict their peers achievement, i.e., they observe $(X_i, X_j, A_i, A_j, \mu, \epsilon_i, \epsilon_j)$, this may not be the case in reality. If, for instance, students observed only their own ϵ_i prior to choosing effort and some mean 0 prior on the effort of peers, ϵ_j would not enter the best response. While these different types of informational assumptions have different implications for identification, the contrast is beyond the scope of the present paper.

ment as a function of individual and peer characteristics is then

$$Y_i^* = \gamma_{0i} + X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j^*\tilde{\gamma}_y + A_i\gamma_a + A_j\tilde{\gamma}_a + \mu\gamma_\mu + \epsilon_i - \epsilon_j\tilde{\gamma}_y. \tag{2.5}$$

The achievement best response takes the same form as the statistical model in equation (2.1), a result that motivates the functional form choices above. Furthermore, the second equality shows that the structure of the marginal effects of the covariates is the same to the ability case in Section 2.1. The only differences are in the constant and the endogenous effect parameter $\tilde{\gamma}_y$, which is a result of the effort best responses and marginal product of effort to achievement.

Table 2: Effort Model Parameter Assumptions

	Structural Parameters		Statistical Model Parameters
(1)	$\alpha_x \geq 0$		without loss of generality
(2)	$\alpha_e > 0$		(2) $\Rightarrow$ Y proxy for effort
(3)	$\beta_e \geq 0$		
(4)	$\tilde{\alpha}_e \geq 0$		
(5i)	$\tilde{\beta}_e \geq 0$		(2)-(4), (5i) $\Rightarrow \tilde{\gamma}_y \geq 0$
(5ii)	$\tilde{\beta}_e < 0$	(a) $\beta_e\alpha_e > \tilde{\beta}_e\tilde{\alpha}_e$	
		(b) $\beta_e\tilde{\alpha}_e \geq -\tilde{\beta}_e\alpha_e$	(2)-(4), (5ii,a&b) $\Rightarrow \tilde{\gamma}_y \geq 0$
		(c) $\beta_e\tilde{\alpha}_e < -\tilde{\beta}_e\alpha_e$	(2)-(4), (5ii,c) $\Rightarrow \tilde{\gamma}_y < 0$
(6)	$\beta_e \geq \tilde{\beta}_e$	$\alpha_e \geq \tilde{\alpha}_e$	(6) $\Rightarrow \tilde{\gamma}_y \leq 1$
(7)	$\alpha_x \geq \tilde{\alpha}_x$		(1), (7), $\tilde{\gamma}_y \leq 1 \Rightarrow \gamma_x \geq 0$
			(1), $\tilde{\gamma}_y \in [0,1]$, (6) $\Rightarrow \tilde{\gamma}_x \lesseqgtr 0$
			(1), $\tilde{\gamma}_y < 0 \Rightarrow \tilde{\gamma}_x$ same sign as $\tilde{\alpha}_x$
(8)	$\frac{\tilde{\alpha}_x}{\alpha_x} \geq \tilde{\gamma}_y$		(1), $\tilde{\gamma}_y \in [0,1]$, (8) $\Rightarrow \tilde{\gamma}_x$ same sign as $\tilde{\alpha}_x$

Similarly to above, Table 2 describes assumptions on the parameters of the structural model and implications for the interpretation of the statistical parameters. I consider first the properties of $\tilde{\gamma}_y$. Assume that $\tilde{\alpha}_e \geq 0$ so that effort and peer effort are weakly complementary inputs to achievement production. This is consistent with the theory that harder working peers create a better learning environment. If $\tilde{\beta}_e \geq 0$, so that i's effort is increasing in peer j's effort, it follows that $\tilde{\gamma}_y \geq 0$. Suppose instead i's effort is decreasing in peer j's effort ($\tilde{\beta}_e < 0$). Then if the intuitively appealing constraint that the effects of i's own effort on utility and achievement exceeds the effect of his peers $\beta_e \alpha_e > \tilde{\beta}_e \tilde{\alpha}_e$, the denominator is positive. If in combination with this assumption $\beta_e \tilde{\alpha}_e \geq -\tilde{\beta}_e \alpha_e$, then $\tilde{\gamma}_y \geq 0$. Otherwise, $\tilde{\gamma}_y < 0$.

It is straightforward to show that $\tilde{\gamma}_y \leq 1$ if $\beta_e \geq \tilde{\beta}_e$, i.e., that the disutility from i's own effort exceeds the (dis)utility derived from conforming to j's effort, and $\alpha_e \geq \tilde{\alpha}_e$, i.e., the marginal effect of i's own effort exceeds that of her peer. Similarly to the ability setting in Section 2.1, this is enough to ensure that shared inputs μ enter the statistical model with the same sign as their marginal product. Furthermore, assuming that $\alpha_x \geq \tilde{\alpha}_x$, then $\gamma_x \geq 0$, taking the same sign as α_x.

The interpretation of $\tilde{\gamma}_x$ also follows similarly to the ability case. A necessary condition for $\tilde{\gamma}_x$ to take the same sign as $\tilde{\alpha}_x$ is that $\frac{\tilde{\alpha}_x}{\alpha_x} \geq \tilde{\gamma}_y$.

If $\tilde{\beta}_e \geq 0$, the larger the individual effect and peer effort spillovers (deriving through the indirect effect of peer effort on i's utility-maximizing effort or the direct effect of peer effort on achievement production), and the smaller the contextual peer effect, the more likely that $\tilde{\gamma}_x$ takes the opposite sign of $\tilde{\alpha}_x$.

If $\tilde{\beta}_e < 0$ and $\beta_e \tilde{\alpha}_e < -\tilde{\beta}_e \alpha_e$, conditions are such that $\tilde{\gamma}_y < 0$. Then, the estimated contextual effect $\tilde{\gamma}_x$ will take the same sign as the structural parameter $\tilde{\alpha}_x$ but will actually overstate the contextual effect, assuming that α_x and $\tilde{\alpha}_x$ have the same sign. Note that this instance of overstating the contextual effect is difficult to justify in the setting where only peer ability matters, unless average peer ability negatively affects achievement production.

As it is written, the current specification is limited because equilibrium effort does not vary explicitly by a student's observable characteristics or classroom inputs. A simple way to incorporate variation in effort across individual and classroom types is to allow the marginal utility of achievement to depend on observable characteristics of the classroom, the individual student, her peers that affect production directly and also potentially other observables Z_i, i.e., $\beta_{yi} = \beta_0 + X_i\beta_x + X_j\tilde{\beta}_x + Z_i\beta_z + \mu\beta_\mu$.

In this case equation (2.5) becomes

$$Y_i^* = \gamma_0 + X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j^*\tilde{\gamma}_y + Z_i\gamma_z + A_i\gamma_a + A_j\gamma_a + \mu\gamma_\mu + \epsilon_i - \epsilon_j\tilde{\gamma}_y, \qquad (2.6)$$

where

$$\begin{aligned}
\gamma_0 &\equiv \beta_0\delta, \\
\delta &\equiv \frac{\alpha_e(\alpha_e^2 - \tilde{\alpha}_e^2)}{\beta_e\alpha_e + \tilde{\beta}_e\tilde{\alpha}_e}, \\
\gamma_x &\equiv \alpha_x - \tilde{\gamma}_y\tilde{\alpha}_x + \delta\beta_x, \\
\tilde{\gamma}_x &\equiv \tilde{\alpha}_x - \tilde{\gamma}_y\alpha_x + \delta\tilde{\beta}_x, \\
\gamma_z &\equiv \delta\beta_z \\
\gamma_\mu &\equiv \alpha_k(1 - \tilde{\gamma}_y + \delta\beta_\mu).
\end{aligned}$$

Maintaining the assumptions that $\alpha_e > 0$ and $\beta_e\alpha_e > -\tilde{\beta}_e\tilde{\alpha}_e$ and that the marginal effect of i's own effort is greater than the effect of her peer's effort ($\alpha_e \geq \tilde{\alpha}_e$), then $\delta \geq 0$. It follows that the addition of peer characteristics in the utility function makes the spillovers from peer characteristics more likely to take the expected sign. Furthermore, allowing for preferences over achievement and effort to vary by observable characteristics highlights an alternative channel through which individual characteristics, such as parental education, may affect achievement production, i.e., through student motivation rather than as a direct input to production.

One particularly useful feature of this extension is that it provides a more explicit role for parents, other home inputs and classroom inputs. However, complementarities between effort and characteristics are likely to enter the best response in other ways. For instance, complementarities between a student's effort and his own or his peers' characteristics in achievement production would produce similar results to the above case. Utility-maximizing effort would then be increasing in own or peer characteristics because the marginal product of effort is increasing in these inputs. Furthermore, an argument could be made that marginal utility is either increasing or decreasing in achievement. If marginal utility is increasing in achievement, then students with "better" X_i would want to exert relatively more effort, which would produce similar results to the above framework. Alternatively, if the marginal utility of achievement is diminishing, this would lead to the opposite effect.

2.3 Discussion

While the model above assumes that there are only 2 students in the classroom for simplicity, it easily extends to the models of the linear-in-means type that are estimated in the literature, where a student's achievement depends on the average of her peers' characteristics and achievement.

Table 3 provides some evidence that the basic theory described above may help explain mixed findings regarding the sign and magnitude of contextual effects in the literature. It is not intended to be a representative survey of the literature, but rather includes studies that report both the estimated effect of average peer achievement and peer characteristics. In particular, the contextual effect parameter appears more likely to take on the "counterintuitive" sign or be smaller in magnitude when the peer achievement effect is larger in magnitude. For instance, Kang (2007) finds that the effect of average peer achievement is 0.304 and the effect of average father's education is negative. Gibbons and Telhaj (2006) find that the effect of average peer achievement is 0.218 and the effect of the percentage of classroom peers who are white is insignificant. **?** find a relatively small effect of peer achievement, 0.072, and the effect of percentage black is negative, -0.137.

The above discussion regarding the interpretation of contextual effects is based on the assumption that peer achievement itself does not matter for production. In a context where we consider only direct externalities of peers to achievement production this seems justified, i.e., the externalities derive through behaviors rather than achievement. In contrast, when students are treated as optimizing agents and choose behaviors based on peers, as suggested in the tracking and acting white literature, there may be a direct role for prior peer achievement in determining effort. For instance, if students are placed with peers who are higher performing, as they observe through knowledge of prior achievement, they could choose to work harder to maintain a certain status in the class. However, as long as this is accompanied by direct externalities from peer effort on production or responses to peer effort, as supported by findings in Lavy and Schlosser (2007), Bishop (2006), Figlio (2007), and Kinsler (2006), among others, similar insights into the interpretation of contextual effects will hold.

3 Implications for Policy: Optimal Grouping

The previous section illustrates how conditioning on peer achievement produces estimates of contextual effects that are difficult to interpret. In this section, I discuss the implications for determining the effects of regrouping students, a central motivation of peer effect studies. The most direct implication is for grouping based on unobservables, such as ability grouping, which I develop in Section 3.1.

However, often policy questions of interest center around observable characteristics of students. For instance, if increased school choice leads to exit of the children with better-educated or higher-income parents, does this hurt the students left behind? Does racial integration improve the performance of black students? In these cases, the reason why the the peer characteristic matters, i.e., whether it's race per sé or the unobservable characteristics correlated with race, may be of secondary importance. With this observation in mind, I then consider in Section 3.2 the potential for reduced form estimates of peer effects (those that attempt to determine the existence of a peer effect rather than the underlying causal mechanism, as clarified below) to inform grouping policies.

Research increasingly recognizes the limitations of the linear-in-means framework and explores how peer effects vary based on student characteristics.[10] Thus, Section 3.3 considers the implications of moving to a more flexible specification. When relevant, in each section I draw contrasts between the framework where the unobservable derives through effort versus ability.

3.1 Grouping on Unobservables

Literature on academic tracking seeks to ascertain the achievement effects of ability grouping versus mixed-ability classrooms.[11] In this context, it may be that policy makers have more information about the student, such as GPA, classroom performance or IQ, than is available in the typical administrative data. Then, it may be natural to condition on peer achievement because the spillovers from peer "ability" themselves are the object of interest.

The real challenge arises if we want to know the separate effect of an observable characteristic, such as race or free/reduced price lunch status, and "ability." Arguably, one of the strengths of peer effects in achievement type studies relative to studies of policy events, such as the historical integration of schools, is that they can isolate the relative importance of different channels of peer influence.

Grouping based on ability that is unobserved to the researcher versus observable achievement suggests both different estimates of the contextual effects and also different contrasts in terms of the changes in peer "ability." For instance, suppose the policy of interest is grouping both on "ability" that is unobserved to the researcher and free/reduced price lunch status. The coefficient on peers receiving free/reduced price lunch includes both a direct

[10] See, for instance, Hoxby and Weingarth (2005), Hoxby (2006), Hanushek et al. (2009), Cooley (2009), Lavy et al. (2008), among others.

[11] See, for example, Slavin (1990), Hallinan (1990), Figlio and Page (2002).

effect ($\tilde{\alpha}_x$) and the indirect effect from conditioning on peer achievement, i.e., both variables are proxying for peer ability. Thus, policy makers would need to be cautious in extending insights from contextual effects parameters in statistical models that control for observable achievement to regrouping policies based on other "ability" measures that are not observed to the researcher.

If, on the other hand, "ability" is the only characteristic of interest and achievement is increasing in ability as in the restricted model presented in Section 2.1, then the parameters of the empirical model can be used to determine whether low-"ability" students are better off in a stratified or a mixed-"ability" setting. A limitation (though probably not a serious one) is that the actual level effect predicted would differ because generally $\tilde{\alpha}_a \neq \tilde{\gamma}_y$ in equations (2.2) and (2.3).

3.2 Grouping on Observable Contextual Effects

Given that many policy questions center around observable peer characteristics, one potential conclusion of the analysis in Section 2 is that it is a mistake to condition on peer achievement. Thus, I turn to the question of grouping on observable peer characteristics. Because the literature on racial composition effects is extensive and of continued concern to policy makers, I center the discussion in this context, though it can certainly be extended to other contextual effects of interest.

Consider a simple setting with two classrooms, $g \in \{c, d\}$, with two students each. Initially the allocation g_0 is such that students $\{1, 2\}$ are in c and $\{3, 4\}$ in d. The characteristics X_i are understood to include a dummy variable for whether the student is white or nonwhite.

The reduced form regression is simply

$$
\begin{aligned}
Y_{1c} &= X_1\pi_x + X_2\tilde{\pi}_x + \mu_c\pi_\mu + \zeta_{1c}, \\
Y_{2c} &= X_2\pi_x + X_1\tilde{\pi}_x + \mu_c\pi_\mu + \zeta_{2c}, \\
Y_{3d} &= X_3\pi_x + X_4\tilde{\pi}_x + \mu_d\pi_\mu + \zeta_{3d}, \\
Y_{4d} &= X_4\pi_x + X_3\tilde{\pi}_x + \mu_d\pi_\mu + \zeta_{4d}.
\end{aligned}
$$

It is a reduced form in the sense that $\tilde{\pi}_x$ does not attempt to distinguish whether the spillover comes from the observed peer characteristic or unobserved peer ability, peer effort, etc. This specification is often referred to as a reduced form of the statistical model in equation (2.1), in which case $\pi_x \equiv \frac{\gamma_x+\tilde{\gamma}_y\tilde{\gamma}_x}{1-\tilde{\gamma}_y^2}$, $\tilde{\pi}_x \equiv \frac{\tilde{\gamma}_x+\tilde{\gamma}_y\gamma_x}{1-\tilde{\gamma}_y^2}$, $\pi_\mu = \tilde{\gamma}_\mu\frac{1+\tilde{\gamma}_y}{1-\tilde{\gamma}_y^2}$, $\zeta_{ik} \equiv \frac{\xi_{ik}+\tilde{\gamma}_y\xi_{jk}}{1-\tilde{\gamma}_y^2}$. The convention in the literature is to describe $\tilde{\pi}_x$ as capturing the *social effect* of peers (the combination of endogenous and contextual effects), following the language introduced in Manski (1993).

I assume that consistent estimates of the reduced form parameters $\pi_x, \tilde{\pi}_x$, are available. To say that $\tilde{\pi}_x$ is identified in this setting, simply means that we are separating an effect of peers from unobserved shared inputs μ_g that could be confounded with peer effects. Studies that rely on random assignment (e.g. Boozer and Cacciola, 2001; Graham, 2008; Sacerdote, 2001) or quasiexperimental designs that approximate random assignment (e.g. Hanushek et al., 2003; Hoxby, 2000; Lavy et al., 2008; Lavy and Schlosser, 2007) most often have the goal of identifying these reduced form parameters.

In the current setting where peer effects are the same across students, average achievement for the population does not change regardless of the grouping. This is because the gains to one group are perfectly offset by the losses to another. Therefore, I focus on equity implications, taking the average achievement of nonwhite students as the outcome of interest. Suppose students $\{1, 2\}$ are nonwhite. Thus, I consider how nonwhite achievement changes when moving from the observed homogeneous arrangement g_0 to an integrated grouping g_1. For instance, grouping students $\{1,3\}$ in c and $\{2,4\}$ in d yields the associated reduced form

$$
\begin{aligned}
Y_{1c} &= X_1\pi_x + X_3\tilde{\pi}_x + \mu_c\pi_\mu + \zeta_{1c},\\
Y_{2d} &= X_2\pi_x + X_4\tilde{\pi}_x + \mu_d\pi_\mu + \zeta_{2d},\\
Y_{3c} &= X_3\pi_x + X_1\tilde{\pi}_x + \mu_c\pi_\mu + \zeta_{3c},\\
Y_{4d} &= X_4\pi_x + X_2\tilde{\pi}_x + \mu_d\pi_\mu + \zeta_{4d}.
\end{aligned}
$$

The average change in achievement for nonwhite students is then

$$
E(Y_1 + Y_2|g_1) - E(Y_1 + Y_2|g_0) = [(X_3 + X_4) - (X_1 + X_2)]\tilde{\pi}_x + E(\pi_\mu(\mu_d - \mu_c)|\vec{X}),
$$

where $\vec{X} = (X_1, X_2, X_3, X_4)$ and I have assumed that $E(\zeta_{ig}|\vec{X}) = 0$ for both classrooms. Under the assumption of random assignment of students to classrooms (and hence $E(\mu_c|\vec{X}) = E(\mu_d|\vec{X}) = 0$), the expected change in achievement depends only on the contextual effects parameter. Thus, reduced form estimates can be applied to predict the effect on average nonwhite achievement of moving from the extreme of a perfectly segregated setting to an integrated setting.

However, random assignment is not the norm. For instance, Clotfelter et al. (2006) find evidence that more highly qualified teachers tend to be matched with more affluent schools or schools with fewer minority students. Because any reassignment of students to classrooms includes a reassignment to teachers, the "ideal" experiment to isolate the effects of regrouping deriving solely through peers might be to fix teacher quality at some value for all classrooms and then determine the effect of reassigning students. Random assignment effectively accomplishes this in expectation.

Yet, there is an important contrast to be drawn between the case where the unobservable peer effect derives through ability (an exogenous effect) versus effort (an endogenous effect) in this setting. As emphasized by Manski (1993) and others, endogenous and contextual peer effects potentially have quite different implications for policy. For instance, suppose we redistribute resources among students within a classroom. In the context where there are endogenous peer effects, this will lead to a *social multiplier* effect, whereby the improvement, or loss, to one student's achievement spills over to other students in the classroom, multiplying the effect of the resource shift (e.g. Manski, 1993; Brock and Durlauf, 2001*b*; Glaeser et al., 2003; Graham, 2008).

In the effort case, social multiplier effects may exist if students respond to teachers. In other words, in the ability setting the classrooms that receive the higher teacher quality will have higher achievement only through the direct effect of the improvement in teacher quality on outcomes. In the effort setting, there is also an indirect effect deriving through the effect of increased teacher quality on student effort and the social multipliers created, i.e., the increase in the effort (achievement) of student 1 leads to an increase for student 2 and vice versa.

Restating this in terms of the parameters of the model, in the reduced form model above, the social multiplier is captured by π_μ. In the ability model in equation (2.3) and the simplest case of the effort model where effort does not vary with teacher inputs, the social multiplier does not exist. This can be seen by plugging in for $\gamma_\mu = (1-\tilde{\gamma}_y)$, so that $\pi_\mu \equiv \tilde{\gamma}_\mu \frac{1+\tilde{\gamma}_y}{1-\tilde{\gamma}_y^2} = \frac{(1-\tilde{\gamma}_y)(1+\tilde{\gamma}_y)}{1-\tilde{\gamma}_y^2} = 1$. However, when effort varies with teacher inputs as in the simple case in equation (2.6) above, $\tilde{\gamma}_\mu = 1-\tilde{\gamma}_y+\delta\beta_k$, and the social multiplier exists, i.e., $\pi_\mu = \frac{1-\tilde{\gamma}_y^2+\delta\beta_k(1-\tilde{\gamma}_y)}{1-\tilde{\gamma}_y^2}$.

Returning now to the case of observational data where teachers are matched to students, with predominantly white schools having relatively higher teacher quality on average than predominantly nonwhite schools. We can approximate $\mu_c^* \equiv \pi_\mu E(\mu_c|X_1, X_2)$ and $\mu_d^* \equiv \pi_\mu E(\mu_d|X_3, X_4)$, for instance, as the average classroom residual, depending on identifying assumptions. This is sufficient for separating out the change in average achievement of nonwhite students deriving from the change in teacher quality under the regrouping, i.e., $\pi_\mu E(\mu_d - \mu_c)|\vec{X}) = \mu_d^* - \mu_c^*$.

These observations extend to the traditional linear-in-means model, which forms the starting point for much of the literature. As described above, reduced form estimates can be used to determine the effect of regrouping students even when unobserved group effects vary systematically with peer group composition. However, this relies heavily on the assumption that, if the social multiplier exists, it is constant across student types. I relax this assumption below.

3.3 Heterogeneous Peer Effects

Both theory and empirical evidence suggest that the simple model that assumes the peer effects are homogeneous across student types may not provide a good approximation of peer effects in practice. In particular, evidence suggests that nonwhites and whites may respond differently to peers, with important implications for the effect of desegregation on the racial achievement gap.[12] For instance, Hanushek et al. (2009) find that the percentage of nonwhite peers has a stronger negative effect on nonwhite students than on whites. This heterogeneity could follow either as a result of heterogeneous contextual effects or endogenous effects or both.

Furthermore, in determining the potential costs of academic tracking, it is often assumed that low-achievers benefit relatively more from being grouped with higher-achieving peers than high achievers. In this context, mixed-ability classes provide the highest average achievement. Extending to the question of desegregation, given that nonwhite students are more highly concentrated in the lower tails of the achievement distribution, integration could also raise average achievement. The literature generally supports the existence of these types

[12]For instance, see Fordham and Ogbu (1986), Cooley (2009), Hanushek et al. (2009), Hoxby and Weingarth (2005), Fryer and Torelli (2005), among others.

of nonlinearities.[13]

When there are heterogeneous peer effects, knowledge of the endogenous peer effect becomes important in many settings. Returning to the previous example, consider the case where there are heterogeneous responses to peer achievement across races, i.e., the statistical model is modified so that

$$Y_{1c} = X_1\gamma_{xn} + X_2\tilde{\gamma}_{xn} + Y_{2c}\tilde{\gamma}_{yn} + \mu_c\gamma_\mu + \xi_{1c},$$

$$Y_{2c} = X_2\gamma_{xn} + X_1\tilde{\gamma}_{xn} + Y_{1c}\tilde{\gamma}_{yn} + \mu_c\gamma_\mu + \xi_{2c},$$

$$Y_{3d} = X_3\gamma_{xw} + X_4\tilde{\gamma}_{xw} + Y_{4d}\tilde{\gamma}_{yw} + \mu_d\gamma_\mu + \xi_{3d},$$

$$Y_{4d} = X_4\gamma_{xw} + X_3\tilde{\gamma}_{xw} + Y_{3d}\tilde{\gamma}_{yw} + \mu_d\gamma_\mu + \xi_{4d},$$

where w denotes white and n nonwhite. The associated reduced form equations are then

$$Y_{1c} = X_1\pi_{xn} + X_2\tilde{\pi}_{\bar{x}n} + \mu_c\pi_{\mu n} + \zeta_{1c},$$

$$Y_{2c} = X_2\pi_{xn} + X_1\tilde{\pi}_{\bar{x}n} + \mu_c\pi_{\mu n} + \zeta_{2c},$$

$$Y_{3d} = X_3\pi_{xw} + X_4\tilde{\pi}_{\bar{x}w} + \mu_d\pi_{\mu w} + \zeta_{3d},$$

$$Y_{4d} = X_4\pi_{xw} + X_3\tilde{\pi}_{\bar{x}w} + \mu_d\pi_{\mu w} + \zeta_{4d},$$

where $\pi_{xr} \equiv \frac{\gamma_{xr}+\tilde{\gamma}_{yr}\tilde{\gamma}_{xr}}{1-\tilde{\gamma}_{yr}^2}$, $\tilde{\pi}_{xr} \equiv \frac{\tilde{\gamma}_{xr}+\tilde{\gamma}_{yr}\gamma_{yr}}{1-\tilde{\gamma}_{yr}^2}$, $\pi_{\mu r} = \frac{1+\tilde{\gamma}_{yr}}{1-\tilde{\gamma}_{yr}^2}$, and $r \in \{n, w\}$. If students are

[13] See, for instance, Cooley (2009), Ding and Lehrer (2007), Hanushek et al. (2003), Hoxby and Weingarth (2005) for evidence of nonlinearities. Ammermueller and Pischke (2009) is an exception.

reassigned to create heterogeneous classes as before, the reduced form would be

$$Y_{1c} = X_1\pi_{xnw} + X_3\tilde{\pi}_{xnw} + \mu_c\pi_{\mu nw} + \zeta_{1c},$$

$$Y_{2d} = X_2\pi_{xnw} + X_4\tilde{\pi}_{xnw} + \mu_d\pi_{\mu nw} + \zeta_{2d},$$

$$Y_{3c} = X_3\pi_{xwn} + X_1\tilde{\pi}_{xwn} + \mu_c\pi_{\mu wn} + \zeta_{3c},$$

$$Y_{4d} = X_4\pi_{xwn} + X_2\tilde{\pi}_{xwn} + \mu_d\pi_{\mu wn} + \zeta_{4d},$$

with $\pi_{xwn} \equiv \frac{\gamma_{xw}+\tilde{\gamma}_{yw}\tilde{\gamma}_{xn}}{1-\tilde{\gamma}_{yw}\tilde{\gamma}_{yn}}$, $\tilde{\pi}_{xwn} \equiv \frac{\tilde{\gamma}_{xw}+\tilde{\gamma}_{yw}\gamma_{xn}}{1-\tilde{\gamma}_{yw}\tilde{\gamma}_{yn}}$, $\pi_{\mu wn} = \frac{1+\tilde{\gamma}_{yw}}{1-\tilde{\gamma}_{yn}\tilde{\gamma}_{yw}}$.

The change in the expected achievement for nonwhites is now

$$E(Y_1 + Y_2|g_1) - E(Y_1 + Y_2|g_0) = (X_1 + X_2)\pi_{xnw} + (X_3 + X_4)\tilde{\pi}_{xnw} - (X_1 + X_2)(\pi_{xn} + \tilde{\pi}_{xn}) + E((\mu_d + \mu_c)\pi_{\mu nw} - 2\mu_c\pi_{\mu n}|\vec{X}).$$

This suggests at least two challenges associated with using the reduced form to estimate effects of regrouping. The first is a support assumption. Given heterogeneous responses to peers, it may not be possible to infer the contextual peer effect of racially mixed classrooms if we only observe homogenous classrooms, i.e., $\pi_{xwn}, \tilde{\pi}_{xwn}, \pi_{nw}, \tilde{\pi}_{xnw}$. Given a sufficiently rich support, however, and a sufficiently flexible estimator, this problem may be mitigated.

Assume that this is the case, and we observe both racially mixed and segregated classrooms. There remains a second problem that derives from the matching of teachers to students, and the fact that unobserved teacher quality in this example may be correlated with the racial composition of the classroom observed in the data. To recover the expected change in average nonwhite achievement, besides the estimated contextual effects it is also necessary to approximate $E((\mu_d + \mu_c)\pi_{\mu nw} - 2\mu_c\pi_{\mu n}|\vec{X})$ using the residuals from the reduced form equations above. Suppose estimates of $\mu_c^* \equiv E(\mu_c\pi_{\mu n}|\vec{X})$ and $\mu_d^* \equiv E(\mu_d\pi_{\mu w}|\vec{X})$ are obtained in the homogenous setting. Similarly for the racially mixed setting, I obtain estimates of $\mu^*_{cnw} \equiv 1/2\mu_c(\pi_{\mu nw} + \pi_{\mu wn})$ and $\mu^*_{dnw} \equiv 1/2\mu_d(\pi_{\mu nw} + \pi_{\mu wn})$. Even though I have effectively assumed that the distribution of teacher quality in the racially mixed setting is the same as in the homogeneous setting, I cannot recover the effect of the reassignment to teachers implicit in the reallocation from the reduced form parameters.

Intuitively, because of assumed matching in the observed data, in the perfectly integrated system, nonwhite (white) students would receive higher (lower) teacher quality on average than the initial observed assignment. If this reallocation to teachers creates social multiplier effects, it is not possible to separate an effect of racial integration from a teacher effect without estimates of the social multiplier (or endogenous) effect. Thus, the ability to apply reduced form estimates to determine the effects of altering peer groups is particularly fragile to the assumption of homogeneous responses to peer achievement.

The key confounding factor is that we cannot obtain a measure of teacher quality that is not a function of the racial makeup of the students in the classroom because of social multiplier effects. As discussed above, this limitation of using the reduced form only applies to settings where social multipliers exist and teachers affect the effort of students. Roderick and Engel (2001), for instance, show that teachers play a significant role in determining students' effort responses to high stakes testing.

4 Identification in the Context of the Model

In the previous section, I have assumed the parameters of interest are identified and contrasted implications for regrouping students in the effort setting when social multiplier exist to the ability setting when social multipliers do not exist. In this section, I contrast the different implications of the two types of unobservables for identification of the endogenous effect, $\tilde{\gamma}_y$. Recall from equation (2.6), the general form of the statistical model where effort choices vary based on characteristics of the student and her peers, i.e.,

$$Y_i = X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j\tilde{\gamma}_y + Z_i\gamma_Z + \mu\gamma_\mu + \xi_i.$$

The challenge for identifying $\tilde{\gamma}_y$ is that Y_i and Y_j are simultaneously determined. One way to address the simultaneity problem is to find an exclusion restriction, a variable that shifts peer achievement (or effort) independently of i's achievement.

First, note that if the spillovers from peer achievement derive only through peer ability, which is exogenous, then no simultaneity problem exists. In this setting, it is possible to identify $\tilde{\gamma}_y$ under the additional assumption that there are no correlated unobservables that simultaneously predict i's achievement and his peers; these are partially captured as shared inputs μ above. This is a particularly strong (and unrealistic) assumption given the difficulty the literature has in measuring teacher quality, an important unobservable (e.g., Hanushek et al., 2005). Given that these assumptions do not hold, it is then natural to ask where potential exclusion restrictions might come from.

It is worth emphasizing that the use of peer achievement to proxy for the unobservable eliminates any potential exclusion restrictions deriving through direct inputs to the achievement production function. This follows because even if we believe the structural production function (2.4) takes the form where, for instance, a student is affected by his own parent's education but not the parental education of his peers (i.e., $\tilde{\alpha}_x = 0$ in equation (2.4) above), parental education would not be a valid exclusion restriction after conditioning on peer achievement. As discussed previously, peer parental education still "affects" achievement in the statistical model as a proxy for peer effort, even in the absence of a direct contextual effect in the structural production function.

However, when effort choices vary based on observable characteristics, as in equation (2.6), this opens an avenue for addressing the simultaneity problem and identifying the endogenous effect parameter. A valid exclusion restriction affects a student's utility-maximizing effort, but does not have a direct effect on achievement production. In this case, a peer's Z's serve as potential exclusions, in the sense that they only affect i's achievement indirectly through her peer's utility maximizing effort. This can be seen formally by solving for Y_j and plugging it back into the statistical model to obtain the following reduced form

$$Y_i = X_i \pi_x + X_j \tilde{\pi}_x + Z_i \pi_z + Z_j \tilde{\pi}_z + \mu \pi_\mu + \tilde{\zeta}_i. \tag{4.1}$$

Here, Z_j is a potential instrument for Y_{jg}, if it satisfies conditional mean independence with μ_g and ξ_{ig} and $\gamma_z \neq 0$.

Note that such an argument cannot be applied to the case where peer achievement spillovers derive only through unobserved exogenous peer attributes or peer ability simply because the students are not maximizing anything.

To consider some examples of potential exclusion restrictions, a policy or program that affects the incentives of some students in the peer group but not others may be useful. Cooley (2009) offers one example–the introduction of student accountability standards, which threaten students with retention if they do not perform above a certain level. Relying on the idea that only "low-achievers" suffer the threat of this policy, the instrument is then the percentage of peers held accountable. Another potential exclusion restriction is a family-level characteristic that affects choice of effort. One example that might not affect achievement production directly could be the presence of a high-achieving sibling. Papers using social networks, peers of peers, can be extended to provide exclusion restrictions that are effectively of this type, i.e., student A affects B and B affects C, but A only affects C indirectly through his effect on B, i.e., A does not enter C's production function directly.[14]

Throughout I have maintained assumptions that generalize to a linear-in-means model of

[14] See Bramoulle et al. (2009) and Laschever (2008) for the broader social interactions context and Giorgi et al. (2007) for the education context.

achievement production with peer spillovers which provides a sort of worst-case scenario for identification, as emphasized by Brock and Durlauf (2001*b*). Cooley (2009) shows how the identifying assumptions can be extended to a more general framework using an effort-type model. The central insight for the exclusion restriction is effectively the same as described above.

5 Dynamics

In this paper, I have focused on a static setting. As much of the literature relies on panel data techniques to identify peer effects, I turn briefly to a discussion of dynamics, both identification in the context of the model and the implications for interpreting contextual peer spillovers. While seemingly worthwhile, a more thorough discussion is well beyond the scope of this paper.

As discussed Section 4, the fact that achievement and peer achievement are simultaneously determined in equilibrium and share common unobserved inputs (such as teacher quality) leads to an identification problem. The literature often addresses this problem by using a lagged measure of peer achievement to proxy for the unobservable.[15] Lagged achievement often corresponds to achievement at the end of the prior academic year, though many of the arguments below will generalize to other settings.

The model presented in Section 2 helps to clarify what part of the unobservable peer effects lagged peer achievement is likely to capture. The use of lagged peer achievement is best justified in a model where students are passive inputs to the production process, i.e., as described in Section 2.1 above where only ability matters. Using once-lagged peer achievement to capture ability relies on the assumption that only peer ability accumulated in the prior year $t-1$ matters for achievement at time t. This does not seem very restrictive

[15] See Hanushek et al. (2003) for discussion.

as it captures all the learning that occurred in the prior academic year, unless one takes into account evidence of differential rates of forgetting across students over summer recess (e.g., Cooper et al., 1996).

In the more common specification, twice-lagged peer achievement serves as the proxy for the unobservable.[16] In this case, the relevant peer ability needs to be from two years prior. Particularly if prior teacher and/or peer quality affect the level of ability acquired and the quality of teachers or peers varies across grades, this may be a less realistic assumption. Of course, a special case where this holds is when ability is a fixed innate characteristics. This is a less desirable assumption in that it effectively negates the role of schools, i.e., there is no human capital accumulation.

Turning to the more general version of the model that incorporates unobservable effort in Section 2.2, the use of lagged measures of peer achievement is more difficult to support in this setting. That said, there are at least two types of assumptions that would work. First, it may be that effort is constant over time (essentially more like a student characteristic) or exogenous and does not respond to current teacher or peer inputs. A slightly less restrictive assumption would be that effort in the current academic year is determined by experiences in the prior academic year but unchanged by current inputs. However, the intuition that teachers in particular may affect student effort is supported by studies such as Roderick and Engel (2001) who show that teachers play an important role in determining how students respond to high stakes testing. Furthermore, Stinebrickner and Stinebrickner (2008) show that ability is not generally a good predictor of effort, but rather that effort varies considerably over time and has a large effect on achievement.

Second, if a student does change effort in response to current inputs, using lagged peer achievement as a proxy still captures the unobservable peer effect if peer effort does not

[16] Two lags are chosen because of controlling for a child's once-lagged achievement in value-added specifications.

matter in the sense that current peers do not affect the effort choice (i.e., $\tilde{\beta}_e = 0$) and peer effort does not affect achievement production (i.e., $\tilde{\alpha}_e = 0$). Figlio (2007), Kinsler (2006) and Lavy and Schlosser (2007) find direct evidence that the disruptive behavior of peers matters for achievement, suggesting that the latter assumption is unlikely to hold.

Given the restrictiveness of these assumptions and evidence in the literature to the contrary, the more likely interpretation of a model that replaces contemporaneous with lagged peer achievement seems to be that of a reduced form specification (as in equation (4.1) above), where the coefficient on prior peer achievement captures both a direct effect of unobserved peer characteristics and the endogenous effect deriving through current peer behavior, or a *social effect*.

5.1 Interpreting Contextual Effects

It is worth emphasizing that the inclusion of lagged peer achievement in the statistical model still leads to similar problems in the interpretation of $\tilde{\gamma}_x$ as discussed in Section 2.1. In fact, if peer groups do not change over time on average, which may be effectively true for many students, and the X_{it} are time-invariant, as is the case with most characteristics available in administrative data, the interpretation of the contextual effect parameter is the same whether conditioning on lagged or contemporaneous achievement. However, if peer groups change over time, the indirect effect of conditioning on lagged achievement is weaker to the extent that average peer characteristics then vary over time. The unobservable peer characteristic is recovered as a function of lagged peer achievement, current peer characteristics and the prior peer characteristics, among other inputs. Thus, even if there is a lot of variation in peer composition in terms of observable characteristics over time, the negative indirect effect deriving through the effect of peers' own characteristics on their lagged achievement will still bias estimates of the contextual peer effect.

Consider a simple example to illustrate. I extend the model in equation (2.2) to include time subscript t and assume that students 1 and 2 are in classroom c at time t, i.e.,

$$Y_{1ct} = X_{1t}\alpha_x + X_{2t}\tilde{\alpha}_x + A_1\alpha_a + A_2\tilde{\alpha}_a + \mu_{ct} + \epsilon_{1ct},$$

$$Y_{2ct} = X_{2t}\alpha_x + X_{1t}\tilde{\alpha}_x + A_2\alpha_a + A_1\tilde{\alpha}_a + \mu_{ct} + \epsilon_{2ct},$$

where I assume that ability is fixed over time for simplicity.

Suppose student 2 was in classroom d in the previous period with a different peer, student 3. Student 2's achievement for the previous period is

$$Y_{2dt-1} = X_{2t-1}\alpha_x + X_{3t-1}\tilde{\alpha}_x + A_2\alpha_a + A_3\tilde{\alpha}_a + \mu_{dt-1} + \epsilon_{2dt-1}$$

Solving for student 2's ability in terms of prior achievement and plugging in for 2's ability in student 1's achievement at time t yields

$$\begin{aligned} Y_{1ct} =& X_{1t}\alpha_x + X_{2t}\tilde{\alpha}_x - X_{2t-1}\alpha_x\frac{\tilde{\alpha}_a}{\alpha_a} + Y_{2dt-1}\frac{\tilde{\alpha}_a}{\alpha_a} - X_{3t-1}\tilde{\alpha}_x\frac{\tilde{\alpha}_a}{\alpha_a} \\ &+ A_1\alpha_a - A_3\frac{\tilde{\alpha}_a^2}{\alpha_a} + \mu_{ct} - \mu_{dt-1}\frac{\tilde{\alpha}_a}{\alpha_a} + \epsilon_{1ct} - \epsilon_{2dt-1}\frac{\tilde{\alpha}_a}{\alpha_a}. \end{aligned}$$

If the students' characteristics are time-invariant as is often the case for the characteristics included in achievement regressions, the contextual effect estimated in the statistical model controlling for lagged peer achievement take the same form as in equation (2.3), i.e., $\tilde{\gamma}_x = \tilde{\alpha}_x - \alpha_x\frac{\tilde{\alpha}_a}{\alpha_a}$. The problem is less severe for time-varying characteristics. However, if peer characteristics are simply highly correlated over time, a similar result on the interpretation of the contextual peer effect will hold.

5.2 Regrouping with Lagged Achievement

Even if controlling for lagged peer achievement does not solve the identification problem or fix the interpretation of contextual effect in the statistical model, it may be useful for informing policy. In some cases lagged achievement may be used explicitly in assigning students to classrooms. For instance, public schools in Wake County, North Carolina, attempted to maintain racial balance by integrating schools based on free/reduced price lunch status and ensuring that low-achievers, as determined by prior year test scores, were distributed somewhat evenly across schools.

This provides an interesting contrast to the grouping on unobservables motivation for controlling for peer achievement as discussed in Section 3.1. In this case, because I know the policy that the district is pursuing, the coefficient on peer free/reduced price lunch status conditional on prior peer achievement is precisely the object of interest to policy makers. In other words, it does not matter that the empirical model does not recover the contextual effect in the structural production function. Of course, if peer effects do in fact differ across student types, the criticism of Section 3.3 still applies.

6 Conclusion

In this paper, I clarify the rationale for endogenous peer effects, i.e., the inclusion of peer achievement in the achievement production function. I take as an underlying premise that peer achievement per sé does not matter in achievement production, but rather serves as a proxy for an unobserved quality of the peer group, as generally argued in the literature. I contrast two types of peer spillovers that peer achievement could capture–unobserved effort and ability. The important distinction is that only the former is truly endogenous, the latter being predetermined.

I highlight three reasons that make standard empirical estimates of peer effects in educational achievement difficult to apply to answer central policy questions regarding the effects of regrouping students. The first lies in the interpretation of the statistical model, i.e., estimates of the spillovers from peer characteristics conditional on peer achievement. Using peer achievement to proxy for unobserved peer "quality" suggests that peer characteristics may appear to be correlated with achievement even if they do not directly affect achievement, but only indirectly as a proxy for peer quality. Furthermore the indirect proxy channel works in opposition to the direct externality that is commonly assumed, suggesting that the intuition that a student should be positively affected by peers with characteristics conducive to achievement may not always bear out in estimates. This finding may help explain mixed evidence in the literature regarding the sign and magnitude of contextual effect.

Second, I show that under reasonable assumptions that are supported elsewhere in the literature, the reduced form estimates of the social effect of peers are not sufficient to determine the effects of regrouping students even when grouping is based on observable characteristics. If unobservable peer effort spillovers exist and effort responds to teacher inputs, reassigning students to teachers creates social multiplier effects. Estimating these social multipliers (or the endogenous effect) is then central to developing viable policy implications of large scale reallocations of students.

Third, if effort spillovers exist, the tendency to ignore the reflection problem, minimizing the importance of simultaneity concerns for identification of peer spillovers, may be misguided. Lagged measures of peer achievement are generally preferred to contemporaneous peer achievement because they are less likely to be correlated with unobserved group effects, such as teacher quality. The model highlights how lagged peer achievement may miss important aspects of the unobservable peer effect. Thus, these estimates cannot be interpreted as causal, limiting their applicability to policy. Though the simultaneity concerns associated with including contemporaneous peer achievement are often thought to be insoluble, the theoretical model further suggests natural exclusion restrictions that permit the identification of endogenous peer effects that are not available in the ability-based framework that is commonly assumed.

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Table 3: Recent Studies on Peer Achievement and Composition.

Studies	Context	Source/ Specification	Peer Achievement	Est. Effect	Contextual Effect	Est. Effect
Cooley (2009)	Public Schools, N. Carolina	Table 3, Col. 1; White Students; IV School-Year FEs	Avg. Reading (T)	.198	% Nonwhite in Classroom	-.085
Gibbons and Telhaj (2006)	Public Schools, England	Table 5, Col. 5	Avg. Key Stage (T-3)	0.218	% White in Grade	-0.004 (insig.)
Hanushek et al. (2003)	Public Schools Texas	Table 2, Col. 3; Student FEs	Avg. Math (T-2)	0.15	% Reduced Price Lunch	0.1
Hanushek et al. (2004)	Public Schools Texas	Table 1, Col. 7; Student FEs	Avg. Math (T-2)	0.01	% Black in Grade	-0.25
Henry and Rickman (2007)	Preschool Georgia	Table 4, Col. 1	Avg. Cognitive (WJ-AP)	0.39	% Black in Classroom	0.12
Kang (2007)	TIMSS countries	Table 5, Col. 7; IV School FEs	Avg. Math (T)	0.304	Avg. Father Education	-.042
Vigdor and Nechyba (2007)	Public Schools N. Carolina	Table 2, Col. 3; School FEs	Avg. Math (T-2)	0.072	% Black in Classroom	-0.137